BOG‘LIQLI TASODIFIY MIQDORLAR UCHUN MARKAZIY LIMIT TEOREMA

Muxtorov Ibrohim G‘aybulla o‘g‘li
National University of Uzbekistan, Tashkent
E-mail: ibrohimmuxtorov199702@gmail.com

ANNOTATSIYA

Bu ilmiy ishda bog‘liqli tasodifiy miqdorlar uchun markaziy limit teorema isbotladim.

Kalit so‘zlar: Markaziy limit teorema, bog‘liqli tasodifiy miqdorlar.

CENTRAL LIMIT THEOREM FOR DEPENDENT RANDOM VARIABLES

ABSTRACT

In this scientific research, I proved central limit theorem for dependent random variables.

Key words: Central limit theorem, dependent random variables

Markaziy Limit Teorema(MLT) ehtimollar nazariyasida muhim hisoblanadi va shu sababli ko’po’rganilgan. Ushbu ishda ba’zi bir dinamiksistemalar uchun markaziy limit teoremani keltiramiz. Aytaylik, (X,T,μ) dinamik sistema bo’lsin(ya’ni T akslantirish (X,μ) Lebeg ehtimollik fazosida o’lchovni saqlasin) va $f \in L^2(\mu)$ markazlashgan, ya’ni $\int_X f d\mu = 0$ bo’lib, f, $f \circ T$, $f \circ T^2, \ldots$ lar bog’liqsiz kema-ketlik bo’lsa, u holda $m \to \infty$ da

$$\mu\left\{ x \in X \mid \frac{S_m f(x)}{\|S_m f\|} < u \right\} \to \Phi(u), \qquad (1)$$

munosabat o'rinli, bu yerda $S_m f = f + f \circ T + ... + f \circ T^{m-1}$, $\|S_m f\|$ son $S_m f$ ning L^2 normasi, ya'ni standart chetlanish va $\Phi(u) = 2(\pi)^{-1/2} \int_{-\infty}^{u} \exp(-\upsilon^2 / 2) d\upsilon$.

Agar (1) shart bajarilsa, biz $f \in L^2(\mu)$ funksiya MLT ni qanoatlantiradi deymiz.

Burishlar uchun MLT. Aytaylik, $X = R / Z$ birlik aylana va μ aylanadagi Lebeg o'lchovli. $\alpha \in X$ irratsional son bo'lsin va $T : X \to X$ akslantirish quyidagicha $T(x) = x + \alpha$ aniqlangan bo'lsin, ya'ni aylanadagi burish bo'lsin.

Ma'lumki, $\int_X f d\mu = 0$ shartni qanoatlantiruvchi ixtiyoriy $f \in L^2(\mu)$ funksiya ushbu $f = \sum_{k=-\infty}^{\infty} b_k g_k$ ko'rinishdagi Fur'e yoyilmasiga ega bo'ladi, bu yerda $b_0 = 0$ va $g_k(x) = e^{2\pi ikx}$ lar xos funksiyalar. Biz qarayotgan f funksiya haqiqiy qiymatli bo'lib, Fur'e koeffisientlari haqiqiy bo'lgani uchun $b_k = b_{-k}$ va $f(x) = \sum_{k=1}^{\infty} 2b_k \cos(2\pi kx)$ bo'ladi.

Endi $\beta = e^{2\pi i\alpha}$ deb olamiz. U holda $Tg_k = \beta^k g_k$ va bundan kelib chiqadiki,

$$S_m f = \sum_{k=-\infty}^{\infty} b_k \frac{1-\beta^{mk}}{1-\beta^k} g_k, \qquad (3)$$

va

$$\sigma_m^2 := \|S_m f\|^2 = \sum_{k=-\infty}^{\infty} 2b_k^2 \left| \frac{1-\beta^{mk}}{1-\beta^k} \right|^2. \qquad (4)$$

Biz quradigan barcha misollar quyidagi tuzilishga ega bo'ladi. Natural sonlarning shunday $J_1, J_2, ...$ qism to'plamlari mavjudki, har bir $n = 1, 2, ...$ uchun

$k_n = \#(J_n)$, $\min(J_{n+1}) > 3\max(J_n)$ va $i, j \in J_n$, $i < j \Rightarrow 3i < j$.

Shu bilan birga, shunday $\varepsilon_n \downarrow 0$ va $\overline{\varepsilon}_n \downarrow 0$ haqiqiy sonlar ketma-ketliklarini olamizki, $j \in J_n$ ekanligidan β^j larning ayalananing birinchi choragida ekanligi va

$$\varepsilon_n > \left|1-\beta^j\right| > \varepsilon_n - \bar{\varepsilon}_n > \frac{\varepsilon}{2}$$

bo'lishi kelib chiqib, $j \in J_n$ va $m\varepsilon_n \le 1$ lar uchun

$$m > \left|\frac{1-\beta^{jm}}{1-\beta^j}\right| > m(1-m\varepsilon_n) \ge 0. \qquad (5)$$

Shu bilan birga, shunday $a_n > 0$ ketma-ketlik olamizki, $n = 1, 2, \ldots$ uchun $j \in J_n \Rightarrow b_j = b_{-j} = a_n$ va $j \notin \bigcup_n J_n \Rightarrow b_j = 0$ o'rinli.

Shuningdek, α irratsional bo'lgani uchun va yuqoridagi shartlarni qanoatlantiruvchi ε_n, $\bar{\varepsilon}$, k_n, a_n lar berilgani sababli J_n ni yuqoridagi kabi aniqlay olamiz va so'ngra agar

$$\sum_{n=1}^{\infty} a_n^2 k_n < \infty \qquad (6)$$

bo'lsa, f ni $f \in L^2(\mu)$ bo'ladigan qilib topamiz.

Bunday aniqlangan f uchun (4) shartni quyidagicha yoza olamiz:

$$\|S_m f\|^2 = \sum_{n=1}^{\infty} 2a_n^2 \sum_{j \in J_n} \left|\frac{1-\beta^{jm}}{1-\beta^j}\right|^2. \qquad (7)$$

Endi $L(n) = 2a_n^2 k_n$ deb olamiz. U holda $4\left|1-\beta^j\right|^{-2} \ge \left|\frac{1-\beta^{jm}}{1-\beta^j}\right|^2$ bo'lgani uchun (5) munosabatdan $S_m f$ ning dispersiyasi uchun, $m\varepsilon_{n_0} \le 1$ shartni qanotlantiruvchi ixtiyoriy n_0 uchun quyidagiga ega bo'lamiz:

$$\begin{aligned} L(n_0)m^2(1-m\varepsilon_{n_0})^2 &\le \|S_m f\|^2 \le \\ &\le \sum_{n<n_0} 16L(n)\varepsilon_n^{-2} + L(n_0)m^2 + m^2 \sum_{n>n_0} L(n). \end{aligned} \qquad (8)$$

Lemma. Har bir γ, $0 < \gamma < 2$ uchun $\int_X f d\mu = 0$ shartni qanoatlantiruvchi shunday $f \in L^2(\mu)$ va $N_0 \subseteq N$ qismiy ketma-ketlik mavjudki, $m \in N_0$ uchun

$$\|S_m f\|^2 \sim m^{2-\gamma}, \qquad (9)$$

munosabat o'rinli va (1) bajariladi, ya'ni $S_m f$ miqdorlar N_0 qismiy ketma-ketlik bo'yicha MLT ni qanoatlantiradi.

Isbot. $L(n)=2^{-\gamma n^2}$, $m=2^{n_0^2}$ va $\varepsilon_n=2^{-n^2-n}$ bo'lsin. U holda yetarlicha katta n_0 larda (8) munosabat

$$m^{-\gamma}m^2(1-2^{-n_0})\le\|S_m f\|^2\le 4\sum_{n<n_0}(2^{-\gamma n^2})(2^{2n^2+2n})+m^{2-\gamma}+m^2\sum_{n>n_0}2^{-\gamma n^2}\le$$

$$\le 8(2^{(2-\gamma)(n_0-1)^2})+2m^2 2^{-\gamma(n_0-1)^2}$$

ko'rinishga kelib, (9) munosabat $N_0=\left\{2^{n^2}\,|\,n=1,2,...\right\}$ qism to'plam uchun bajariladi.

$S_m f$ ning dispersiyasi haqidagi bu baho shuni ko'rsatadiki, $S_m f$ yig'indi L^2 da o'zining J_{n_0} ga mos bloklardan hosil qilingan Fure qatorlarining hadlari orqai yaxshi approksimatsiya qilinadi.

Shunday qilib, (1) munosabatni $m=2^{n_0^2}\to\infty$ da isbotlash uchun $S_m f$ ni quyidagi bilan almashtiramiz:

$$a_{n_0}\sum_{\pm j\in J_{n_0}}\left(\frac{1-\beta^{jm}}{1-\beta^{j}}\right)g_j\,.$$

Ma'lum k_{n_0} da, juda kichik $\bar{\varepsilon}_{n_0}$ uchun $\left|1-\beta^j\right|$ sonni ε_{n_0} ga teng deb faraz qilish mumkin va kichik L^2 xatolik bilan

$$\frac{1-\beta^{jm}}{1-\beta^{j}}g_j+\frac{1-\beta^{-jm}}{1-\beta^{-j}}g_j$$

ifodani

$$C_{n_0}\cos(2\pi jx)+D_{n_0}\sin(2\pi jx)$$

ifoda bilan almashtirishimiz mumkin, bu yerda C_{n_0} va D_{n_0} lar faqatgina ε_{n_0} ga bog'liq.

Yuqoridagi (1) munosabatda $S_m f/\|S_m f\|$ ifodani ushbu

$$\frac{1}{A_{n_0}\sqrt{k_{m_0}}}\sum_{j\in J_{n_0}}\left(C_{n_0}\cos(2\pi jx)+D_{n_0}\sin(2\pi jx)\right)$$

ifoda bilan almashtiramiz, bu yerda $A_{n_0}=\frac{\sqrt{C_{n_0}^2+D_{n_0}^2}}{2}$. Oson ko'rish mumkinki, biz k_{n_0} ni yetarlicha katta qilib tanlay olganimiz sababli oxirgi ifoda standart normal taqsimotga taqsimot bo'yicha yaqinlashadi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI: (REFERENCES)

1. Robert Burton, Manfred Denker, On the Central Limit Theorem for dynamical systems,Transaction of the American Mathematical Society, Volume 302, Number 2, August 1987, 715-718.

2. M.Weber, On the CLT for means under the rotation action, Society for Industrial and Applied Mathematics, Theory Probab.Appl.Vol.50. No.4. pp.631-649.

3.Sh.Q. Formanov, Ehtimolliklar nazariyasi, Toshkent, "Universitet" 2014.

4. T.De La Rue, S.Ladouceur, G.Peskirand M.Weber, On the central limit theorem for aperiodic dynamical systems and applications, Theory Probab. Math. Statist., 57(1998), pp.149-169.

5. R.Salem and A.Zygmund, On lacunary trigonometric series, Proc. Nat. Acad. Sci. U.S.A. 33(1947), 333-338; II ibid. 34(1948), 54-62.

6. M. Ledoux and M. Talagrand, Probability in Banach spaces, Springer-Verlag, Berlin, 1990.