MARKAZIY LIMIT TEOREMA O‘RINLI BO‘LADIGAN FUNKSIYALAR SINFI

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ANNOTATSIYA

Bu ilmiy ishda bog‘liqli tasodifiy miqdorlar uchun markaziy limit teorema isbotladim.

Kalit so‘zlar: Markaziy limit teorema, bog‘liqli tasodifiy miqdorlar.

A CLASS OF FUNCTIONS FOR WHICH THE CENTRAL LIMIT THEOREM HOLDS

ABSTRACT

In this scientific research, I proved central limit theorem for dependent random variables.

Key words: Central limit theorem, dependent random variables

Markaziy limit teorema ehtimollar nazariyasida muhim ahamiyatga ega.
Teorema. Faraz qilaylik, ushbu

$$\lim_{j\to\infty}\frac{\varepsilon_j+|\rho_j|}{\left(\sum_{k>j}|\rho_j|^2\right)^{1/2}}=0$$

shart bajarilsin. U holda F uchun MLT o‘rinli bo‘ladi.

Lemma.

(a) Aytaylik, $\{z_k, k\in\Delta\}$ komleks sonlar quyidagi shartni qanoatlantirsin:

$$h_0=\sum_{k\in\Delta}e^{-\frac{z_k^2}{2}}\left\|\left[\frac{1}{4}e^{\frac{z_k^2}{2}}|z_k|^4+\frac{1}{3}\frac{|z_k|^3}{|1+z_k|}e^{|z_k|}\right]\right\|\le\frac{1}{2}.$$

U holda

$$\left|\frac{\prod_{k\in\Delta}e^{z_k}}{\prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}}}-1\right|\le 2h_0.$$

(b) Faraz qilaylik, z_k lar sof mavhum kompleks sonlar bo‘lib,

$$h=\sum_{k\in\Delta}\left[\frac{1}{4}e^{z_k^2}|z_k|^4+\frac{1}{3}\frac{|z_k|^3}{\sqrt{1+|z_k|^2}}e^{|z_k|+\frac{z_k^2}{2}}\right]\le\frac{1}{2},$$

bo‘lsin. U holda

$$\left|\prod_{k\in\Delta}e^{z_k}-\prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}}\right|\le 2h.$$

Isbot. $a_k=\dfrac{e^{z_k}}{(1+z_k)e^{\frac{z_k^2}{2}}}-1,\ k\in\Delta$ deb olamiz. Ixtiyoriy z kompleks son uchun o‘rinli bo‘lgan ushbu

$$z:\left|e^{z_k}-(1+z_k)e^{\frac{z_k^2}{2}}\right|\le\frac{1}{4}|1+z||z|^4e^{\frac{|z^2|}{2}}+\frac{1}{3}|z|^3e^{|z|},$$

tengsizlikdan foydalanib, quyidagiga ega bo‘lamiz:

$$|a_k|\le\left|e^{-\frac{|z_k|^2}{2}}\right|\left\|\left[\frac{1}{4}e^{\frac{z_k^2}{2}}|z_k|^4+\frac{1}{3}\frac{|z_k|^3}{|1+z_k|}e^{|z_k|}\right]\right.$$

Natijada, $\sum_{k\in\Delta}|a_k|=h\le\frac{1}{2}$, va

$$\left|\frac{\prod_{k\in\Delta}e^{z_k}}{\prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}}}-1-\sum_{k\in\Delta}a_k\right|\le\frac{h^2}{1-h},$$

munosabat o'rinli. Bundan (a) tasdiq kelib chiqadi.

Endi (b) tasdiqni isbotlaymiz. Ta'kidlaymizki,

$$|1+z_k|\left|e^{\frac{z_k^2}{2}}\right| = |1+i|z_k|\left|e^{-\frac{|z_k|^2}{2}}\right| \le 1,$$

o'rinli va natijada $\prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}} \le 1$ bo'lishini hosil qilamiz. (a) tasdiq shartidagi tengsizlikning ikkala tomonini $\prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}}$ ga ko'paytirib, quyidagini olamiz:

$$\left|\prod_{k\in\Delta} e^{z_k} - \prod_{k\in\Delta}(1+z_k)e^{\frac{z_k^2}{2}}\right| \le 2\sum_{k\in\Delta}\left[\frac{1}{4}e^{|z_k|^2}|z_k|^4 + \frac{1}{3}\frac{|z_k|^3}{\sqrt{1+|z_k|^2}}e^{|z_k|+\frac{z_k^2}{2}}\right] = 2h.$$

Teoremaning isboti. Ta'kidlaymizki,

$$S_N F(t) = N\sum_{k=1}^{\infty}\mathrm{Re}\left[\rho_k \overline{V_N(l_k\theta)e_{l_k}(t)}\right].$$

Quyidagi belgilashlarni kiritamiz: ixtiyoriy $j \ge 1$, $N \in I_j$ va $t \in X$ uchun

$$\breve{S}_N F(t) = N\sum_{k>j}\mathrm{Re}\left[\rho_k \overline{V_N(l_k\theta)e_{l_k}(t)}\right],$$

$$\hat{S}_N f(t) = N\sum_{k\le j}\mathrm{Re}\left[\rho_k \overline{V_N(l_k\theta)e_{l_k}(t)}\right], \qquad (3.2.1)$$

$$\breve{S}_N F(t) = S'_N F + s_N F,\; S'_N F = N\sum_{k<j}\mathrm{Re}\left[\rho_k \overline{V_N(l_k\theta)e_{l_k}(t)}\right],$$

$$F_j(t) = \sum_{k>j}\mathrm{Re}\left[\rho_k \overline{e_{l_k}(t)}\right],\; c_j = \left(\sum_{k>j}|\rho_k|^2\right)^{1/2},\; \sigma_N = Nc_j.$$

Bundan kelib chiqadiki,

$$\left\|\hat{S}_N f - NF_j\right\|^2 = N^2\sum_{k>j}|\rho_k|^2|V_N(l_k\theta)-1|^2 \le 4N^2\varepsilon_{j+1}^2 c_j^2,$$

$$\left\|S'_N\right\|^2 \le 4N^2\varepsilon_j^2 c_0^2, \qquad \|s_N\| \le N|\rho_j|.$$

Bundan, $N \to \infty$ da

$$\left\| \frac{S_N F}{\sigma_N} - \frac{F_j}{c_j} \right\| \le \left\| \frac{\breve{S}_N F}{\sigma_N} - \frac{F_j}{c_j} \right\| + \left\| \frac{\hat{S}_N F}{\sigma_N} \right\| \le 2\varepsilon_{j+1} + \frac{2\varepsilon_j + |\rho_j|}{c_j} \to 0.$$

Shu bilan birga, uchburchak tengsizligidan

$$\lim_{N\to\infty} \frac{\|S_N F\|_2}{\sigma_N} = 1.$$

Ushbu $|e^{iu} - e^{iv}| \le 2|\sin((v-u)/2)| \le |v-u|$ elementar tengsizlikdan foydalanib, $N \to \infty$ da quyidaglarni hosil qilamiz:

$$\left| \int_0^1 \left(\exp\left(i\lambda \frac{S_N F(t)}{\sigma_N} \right) - \exp\left(i\lambda \frac{F_j(t)}{c_j} \right) \right) dt \right| \le |\lambda| \left\| \frac{S_N F}{\sigma_N} - \frac{F_j}{c_j} \right\|_1 \le$$

$$\le |\lambda| \left\{ 2\varepsilon_{j+1} + \frac{2\varepsilon_j + |\rho_j|}{c_j} \right\} \to 0.$$

Aytalik, $\kappa : \mathrm{N} \to \mathrm{N}$ akslantirish ushbu $\sum_{k>\kappa(j)} |\rho_k|^2 / c_j^2 \le \varepsilon_j^2$, $j \ge 1$ shartni qanoatlantiruvchi biror o‘suvchi akslantirish bo‘lsin va $\Delta_j = [j, \kappa(j))$, $F_j^{'} = \sum_{j<k\le\kappa(j)} \mathrm{Re}\left[\rho_k \overline{e_{l_k}(t)}\right]$, $j \ge 1$ deb olamiz. U holda, $j \to \infty$ da

$$\left| \int_0^1 \left(\exp\left(i\lambda \frac{F_j(t)}{b_j} \right) - \exp\left(i\lambda \frac{F_j^{'}(t)}{b_j} \right) \right) dt \right| \le |\lambda| \frac{\|F_j - F_j^{'}\|_1}{c_j} \le |\lambda| \varepsilon_j \to 0.$$

Endi $k > j$ va $t \in X$ uchun $z_k^j = \mathrm{Re}\left[\rho_k \overline{e_{l_k}(t)}\right] / c_j$ deb olamiz. U holda shartga ko‘ra $j \to \infty$ uchun

$$\sup_{k>j} \sup_{t\in X} |z_k^j(t)| \le \sup_{k>j} \frac{|\rho_k|}{c_k} \to 0.$$

Aytaylik, $\Lambda \in N$ bo‘lsin. U holda faqatgina Λ ga bog’liq bo‘lgan biror chekli butun $J = J(\Lambda)$ uchun

$$\sup_{k>j} \sup_{t\in X} |z_k^j(t)| \le \frac{1}{2}.$$

h miqdor quyidagicha baholanadi:

$$h \le \Lambda^3 \left[\frac{1}{8} e^{1/4} + \frac{\sqrt{2}}{3} e^{1/2+1/8} \right] \sum_{k\in\Delta_j} \left| z_k^j(t) \right|^3 \le C\Lambda^3 \frac{\sum_{k>j} |\rho_k|^3}{\left(\sum_{k>j} |\rho_k|^2 \right)^{3/2}} \le C\Lambda^3 \sup_{k>j} \frac{|\rho_k|}{c_k},$$

hamda $j \to \infty$ da nolga intiladi. Kerak bo'lgan hollarda J ni qayta aniqlash orqali, barcha $t \in X$ va $j \ge J$ lar uchun quyidagiga ega bo'lamiz:

$$\left| \exp\left(i\lambda \frac{F_j^{'}(t)}{b_j} \right) - \prod_{k\in\Lambda_j} z_k^j(t) \exp\left(\frac{z_k^j(t)^2}{2} \right) \right| \le C\Lambda^3 \sup_{k>j} \frac{|\rho_k|}{c_k}.$$

Oxirgi munosabatning ikkala tomonini X to'plamda integrallab, $j \ge J$ uchun

$$\left| \int_0^1 \left(\exp\left(i\lambda \frac{F_j^{'}(t)}{b_j} \right) - \prod_{k\in\Lambda_j} z_k^j(t) \exp\left(\frac{z_k^j(t)^2}{2} \right) \right) \right| \le C\Lambda^3 \sup_{k>j} \frac{|\rho_k|}{c_k},$$

bo'lishini hosil qilamiz.

Endi, ixtiyoriy butun $j \ge 1$ uchun

$$\prod_j (\lambda, t) = \prod_{k\in\Lambda_j} (1 + i\lambda z_k^j(t)),\ B_j(t) = \sum_{k\in\Delta_j} \left(z_k^j(t) \right)^2,$$

deb olamiz. U holda

$$\int_0^1 \prod_j (\lambda, t) dt = 1,$$

ga ega bo'lamiz. Haqiqatan ham,

$$\prod_j \left(1 + i\lambda \left[\beta_k^1 \cos 2\pi l_k t + \beta_k^2 \sin 2\pi l_k t \right] \right),$$

ko'paytma 1 va $\cos 2\pi(l_{k_1} \pm ... \pm l_{k_r})$ yoki $\sin 2\pi(l_{k_1} \pm ... \pm l_{k_r})$ larning yig'indisi ko'rinishida ifodalanadi. Farazimizga ko'ra, $\{l_k, k \in N\}$ ketma-ketlik r-lukunar ketma-ketlik($r \ge 3$) edi. U holda n son uchun $n = l_{k_1} \pm ... \pm l_{k_r}$ ko'rinishdagi yoyilmaning yagonaligidan $\int_0^1 \prod_j (\lambda, t) dt = 1$ bo'lishini hosil qilamiz.

Quyidagi ko‘paytmalarni yozishimiz mumkin:

$$\left|\int_0^1 \prod_j (\lambda,t)\cdot \exp\left(-\frac{\lambda^2 B_j(t)}{2}\right)dt - \exp\left(-\frac{\lambda^2}{4}\right)\right| =$$

$$= \left|\int_0^1 \prod_j (\lambda,t)\left[\exp\left(-\frac{\lambda^2 B_j(t)}{2}\right) - \exp\left(-\frac{\lambda^2}{4}\right)\right]dt\right| \le$$

$$\le \int_0^1 \left|\prod_j (\lambda,t)\right|\left|\exp\left(-\frac{\lambda^2 B_j(t)}{2}\right)dt - \exp\left(-\frac{\lambda^2}{4}\right)\right|dt\,.$$

Shuning uchun,

$$\left|\prod_j (\lambda,t)\right| = \prod_{j<k\le\kappa(j)} \left(1+|\lambda|^2\left|z_k^j(t)\right|^2\right)^{1/2} \le \exp\left(\frac{\lambda^2 B_j(t)}{2}\right).$$

Natijada,

$$\int_0^1 \left|\prod_j (\lambda,t)\right|\left|\exp\left(-\frac{\lambda^2 B_j(t)}{2}\right)dt - \exp\left(-\frac{\lambda^2}{4}\right)\right|dt \le$$

$$\le \int_0^1 \left|1-\exp\left(-\frac{\lambda^2}{2}\left[\frac{1}{2}-B_j(t)\right]\right)\right|dt.$$

Biroq

$$B_j(t) = \frac{1}{c_j^2}\left\{\sum_{k\in\Delta_j}\frac{(\beta_k^1)^2+(\beta_k^2)^2}{2} - \sum_{k\in\Delta_j}\frac{(\beta_k^1)^2-(\beta_k^2)^2}{2}\cos 4\pi l_k t + \sum_{k\in\Delta_j}\beta_k^1\beta_k^2 \sin 4\pi l_k t\right\},$$

yoyilma o‘rinli. Endi

$$C_j(t) = \frac{1}{2} - B_j(t) - c_j^{-2}\sum\nolimits_{k>k_j}\left((\beta_k^1)^2+(\beta_k^2)^2\right)/\,2\,,$$

deb belgilasak, $0\le B_j(t)\le 2$ ekanligidan

$$\int_0^1 \left|1-\exp\left(-\frac{\lambda^2}{2}\left[\frac{1}{2}-B_j(t)\right]\right)\right|dt \le \frac{\Lambda^2}{2}e^{3\Lambda^2/4}\left[\frac{1}{c_j^2}\sum_{k>\kappa(j)}\frac{(\beta_k^1)^2+(\beta_k^2)^2}{2} + \left\|C_j\right\|_2\right].$$

Endi, shartga ko‘ra j cheksizlikka intilgani uchun,

$$\left\|C_j\right\|_2^2 \le \frac{1}{4}\sum_{k\in\Delta_j}\frac{\left|\rho_k\right|^4}{c_j^4} \le \frac{1}{4}\sup_{k>j}\frac{\left|\rho_k\right|^2}{c_k^2} \to 0,$$

munosabat o'rinli bo'lishini tekshirishimiz kerak. Barcha baholarni jamlasak, $|\lambda| \le \Lambda$ va $j \ge J = J(\Lambda)$, $N \in I_j$ uchun N cheksizlikka intilganda

$$\left|\int_0^1 \exp\left(i\lambda\frac{S_N F(t)}{\sigma_N}\right)dt - \exp\left(-\frac{\lambda^2}{4}\right)\right| \le \Lambda\left\{2\varepsilon_{j+1} + \frac{2\varepsilon_j + \left|\rho_k\right|}{c_j}\right\} + \varepsilon_j\Lambda +$$

$$+C\Lambda^3\sup_{k>j}\frac{\left|\rho_k\right|}{c_k} + C\Lambda^2\exp\left(\frac{3}{4}\Lambda^2\right)\sup_{k>j}\frac{\left|\rho_k\right|^2}{c_k^2} \to 0,$$

bo'lishi kelib chiqadi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI: (REFERENCES)

1. Robert Burton, Manfred Denker, On the Central Limit Theorem for dynamical systems,Transaction of the American Mathematical Society, Volume 302, Number 2, August 1987, 715-718.

2. M.Weber, On the CLT for means under the rotation action, Society for Industrial and Applied Mathematics, Theory Probab.Appl.Vol.50. No.4. pp.631-649.

5. R.Salem and A.Zygmund, On lacunary trigonometric series, Proc. Nat. Acad. Sci. U.S.A. 33(1947), 333-338; II ibid. 34(1948), 54-62.

6. M. Ledoux and M. Talagrand, Probability in Banach spaces, Springer-Verlag, Berlin, 1990.