MAKSIMAL NOCHIZIQLI INTEGRO-DIFFERENSIAL TENGLAMALAR SISTEMASI UCHUN CHEGARAVIY MASALA

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ANNOTATSIYA

Impulsiv effektlar va maksimallarga ega oddiy integro-differensial tenglamalarning birinchi tartibli sistemasi uchun nolokal chegaraviy masala o‘rganildi. Chegaraviy masala integral shart bilan beriladi. Ketma-ket yaqinlashish usuli, uni qisqartirib aks ettirish usuli bilan birgalikda qo‘llaniladi. Chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan. Yechimlarning chegaraviy shartning o‘ng tomoniga uzluksiz bog‘liqligi ko‘rsatilgan.

Kalit so‘zlar: impulsiv integrodifferensial tenglamalar, nolokal chegaraviy shart, ketma-ket yaqinlashishlar, yechimning mavjudligi va yagonaligi, yechimning uzluksiz bog‘liqligi.

Masalaning qo‘yilishi

$[0,T]$ segmentda chiziqli bo‘lmagan integro-differensial tenglamaning quyidagi birinchi tartibli sistemasini ko‘rib chiqamiz

$$x'(t)=f\left(t,x(t),\int_0^T\Theta\left(t,s,\max\left\{x(\tau)\,|\,\tau\in\left[\lambda_1 s;\lambda_2 s\right]\right\}\right)ds\right),\ t\neq t_i,\ i=1,2,...,p \quad (1)$$

nolokal chegaraviy shart

$$Ax(0)+\int_0^T K(t,s)\,x(s)\,ds=B(t) \quad (2)$$

va nochiziqli impulsiv ta’sir

$$x\left(t_i^+\right)-x\left(t_i^-\right)=I_i\left(x\left(t_i\right)\right),\quad i=1,2,...,p, \quad (3)$$

berilgan bo‘lsin, bu yerda $0=t_0<t_1<...<t_p<t_{p+1}=T$, $A\in R^{n\times n}$ matritsa, $K(t,s)$ $n\times n-$ o‘lchovli matritsa funktsiyasi va $\det Q(t)\neq 0,\ Q(t)=A+\int_0^T K(t,s)ds,$ $f:[0,T]\times R^n\times R^n\to R^n,\ \ \Theta:[0,T]^2\times R^n\to R^n,\ \ I_i:R^n\to R^n$ berilgan funksiyalar;

$0<\lambda_1<\lambda_2<1,\quad x\left(t_i^+\right)=\lim\limits_{h\to 0^+} x\left(t_i+h\right),\quad x\left(t_i^-\right)=\lim\limits_{h\to 0^-} x\left(t_i-h\right)$ mos ravishda $x(t)$ funksiyaning $t=t_i$ nuqtadagi o‘ng tomonli va chap tomonli limitlari.

$C\left([0,T],R^n\right)$ fazo $x(t)$,uzluksiz vektor funksiyalardan tashkil topgan Banax fazosini bildiradi, aniqlanish sohasi $[0,T]$, qiymatlar sohasi R^n va normasi

$$\| x\|=\sqrt{\sum_{j=1}^{n}\max_{0\le t\le T}\left| x_j(t)\right|}.$$

$PC\left([0,T],R^n\right)$ chiziqli vektor fazo quyidagicha bo‘ladi:

$$PC\left([0,T],R^n\right)=\left\{x:[0,T]\to R^n;\ x\left(t\right)\in C\left(\left(t_i,t_{i+1}\right],R^n\right),\ i=1,\dots,p\right\},$$

bu yerda $x\left(t_i^+\right)$ va $x\left(t_i^-\right)$ $(i=0,1,\dots,p)$ mavjud va chegaralangan; $x\left(t_i^-\right)=x\left(t_i\right)$.

$PC\left([0,T],R^n\right)$ chiziqli vektor fazoning Banax fazosidagi normasi quyidagicha:

$$\| x\|_{PC}=\max\left\{\| x\|_{C((t_i,t_{i+1}])},\ i=1,2,\dots,p\right\}.$$

Masalani yechilishi. $x(t)\in PC\left([0,T],R^n\right)$ funksiya barcha $t\in[0,T],\ t\ne t_i,\ i=1,2,\dots,p$ lar uchun (1) integro-differensial tenglamani qanoatlantiradi, (2) nolokal integral shartni va $t=t_i\ i=1,2,\dots,p,\ 0<t_1<t_2<\dots<t_p<T$ lar uchun (3) nochiziqli limit shartini qanoatlantiradi.

Integral tenglamaga keltirish

$x(t)\in PC\left([0,T],R^n\right)$ funksiya (1)-(3) nolokal chegaraviy masalaning yechimi bo‘lsin. U holda (1) tenglamani $t\in\left(0,t_{i+1}\right]$, intervalda integrallash orqali quyidagi tenglikka ega bo‘lamiz:

$$\int_0^t f(s,x,y)\,ds=\int_0^t x'(s)\,ds=\left[x\left(t_1\right)-x\left(0^+\right)\right]+\left[x\left(t_2\right)-x\left(t_1^+\right)\right]+\dots+\left[x(t)-x\left(t_i^+\right)\right]=$$

$$=-x(0)-\left[x\left(t_1^+\right)-x\left(t_1\right)\right]-\left[x\left(t_2^+\right)-x\left(t_2\right)\right]-\dots-\left[x\left(t_i^+\right)-x\left(t_i\right)\right]+x(t).$$

(3) shartni hisobga olib, oxirgi tenglikni quyidagicha yozamiz:

$$x(t) = x(0) + \int_0^t f(s,x,y)\,ds + \sum_{0<t_i<t} I_i\left(x(t_i)\right). \tag{4}$$

(4) dagi $x(t) \in PC\left([0,T], R^n\right)$ funksiya (2) chegaraviy shartni qanoatlantirishini ko‘rishimiz mumkin:

$$\left[A + \int_0^T K(t,s)ds\right] x(0) =$$

$$= B(t) - \int_0^T K(t,s) \int_0^s f(\theta,x,y)\,d\theta ds - \int_0^T K(t,s) \sum_{0<t_i<t} I_i\left(x(t_i)\right) ds. \tag{5}$$

$\det Q(t) = \det\left[A + \int_0^T K(t,s)ds\right] \neq 0$ bo‘lgani tufayli (5) tenglikni qayta yozamiz

$$x(0) = Q^{-1}(t)\left[B(t) - \int_0^T K(t,s) \int_0^s f(\theta,x,y)\,d\theta ds - \int_0^T K(t,s) \sum_{0<t_i<t} I_i\left(x(t_i)\right) ds\right]. \tag{6}$$

(6) tenglikni (4) tenglikga qo‘yib, quyidagi tenglikka ega bo‘lamiz:

$$x(t) = Q^{-1}(t)\left[B(t) - \int_0^T K(t,s) \int_0^s f(\theta,x,y)\,d\theta ds - \int_0^T K(t,s) \sum_{0<t_i<t} I_i\left(x(t_i)\right) ds\right] +$$

$$+ \int_0^t f(s,x,y)ds + \sum_{0<t_i<t} I_i\left(x(t_i)\right). \tag{7}$$

Quyidagi tengliklarni hisobga olib

$$\int_0^T K(t,s) \int_0^s f(\theta,x,y)\,d\theta ds = \int_0^T \int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds,$$

$$\int_0^T K(t,s) \sum_{0<t_i<t} I_i\left(x(t_i)\right) ds = \sum_{0<t_i<T} \int_{t_i}^T K(t,s)\,ds\, I_i\left(x(t_i)\right),$$

(7) tenglikdan quyidagi tenglikka ega bo‘lamiz:

$$x(t) = Q^{-1}(t)B(t) - Q^{-1}(t) \int_0^T \int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds -$$

$$-Q^{-1}(t)\sum_{0<t_i<t}\int_{t_i}^{T}K(t,s)\,ds\,I_i\left(x\left(t_i\right)\right)+\int_0^t f(s,x,y)\,ds+\sum_{0<t_i<t}I_i\left(x\left(t_i\right)\right). \qquad (8)$$

(8) tenglikda ba’zi soddalashtirishlardan so‘ng quyidagi tengliklarga ega bo‘lamiz:

$$\int_0^t f(s,x,y)\,ds-Q^{-1}(t)\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds=$$

$$=Q^{-1}(t)\int_0^t\left(A+\int_0^s K(t,\theta)d\theta\right)f(s,x,y)\,ds-$$

$$-Q^{-1}(t)\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds; \qquad (9)$$

$$\sum_{0<t_i<t}I_i\left(x\left(t_i\right)\right)-Q^{-1}(t)\sum_{0<t_i<T}\int_{t_i}^{T}K(t,s)\,ds\,I_i\left(x\left(t_i\right)\right)=$$

$$=Q^{-1}(t)\sum_{0<t_i<t}\left(A+\int_0^{t_i}K(t_i,s)ds\right)I_i\left(x\left(t_i\right)\right)-\sum_{t<t_{i+1}<T}Q^{-1}(t_i)\int_{t_i}^{T}K(t_i,s)\,ds\,I_i\left(x\left(t_i\right)\right). \qquad (10)$$

(9) va (10) tengliklarni (8) tenglikka qo‘yib quyidagi tenglikga ega bo‘lamiz:

$$x(t)=J(t;x)\equiv Q^{-1}(t)B(t)+\sum_{0<t_i<t}G\left(t,t_i\right)I_i\left(x\left(t_i\right)\right)+$$

$$+\int_0^T G(t,s)\, f\left(s,x(s),\int_0^T\Theta\left(s,\theta,\max\left\{x(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)ds \qquad (11)$$

bu yerda

$$G(t,s)=\begin{cases}Q^{-1}(t)\left(A+\int_0^s K(t,\theta)d\theta\right), & 0\le s\le t,\\ -Q^{-1}(t)\int_s^T K(t,\theta)d\theta, & t<s\le T.\end{cases}$$

$$t\in\left(t_i,t_{i+1}\right],\ i=0,1,\dots,p,.$$

(11) tenglama (1)-(3) masalani qanoatlantirishini tekshirish qiyin emas.

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