AYRIM IRRATSIONAL KO‘RINISHDAGI INTEGRALLARNI EYLER ALMASHTIRISHLARI YORDAMIDA RATSIONALLASHTIRISH

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Annotatsiya : $\int R\left(x, \sqrt{ax^2+bx+c}\right)dx$ ko‘rinishidagi integrallarni hisoblashda biz avvalo bu integral ostidagi ifodani ratsional funksiyaga keltirib olishimiz zarur. Buning uchun Eylerning 3 ta almashtirishi bizga yordam beradi. Quyida biz shu almashtirishlarni misollarda ko‘rib chiqamiz.

Kalit so‘zlar: Irratsional integral, ratsional funksiya, Eyler almashtirishlari, kvadrad uchhad.

Bizga irratsional ko‘rinishdagi

$$\int R(x, \sqrt{ax^2+bx+c})dx \qquad (1)$$

integral berilgan bo‘lsin. Ushbu integralni quyidagi uchta almashtirish yordamida ratsional funksiya integraliga keladi.Odatda bu almashtirishlarni Eyler almashtirishlari deymiz. (1) integralda a, b, c -haqiqiy sonlar bo‘lib, ax^2+bx+c kvadrat uchhad teng ildizlarga ega emas.

1-holat $a > 0$ bo‘lsin.

(1) integralda ushbu

$$t = \sqrt{a}x + \sqrt{ax^2+bx+c} \qquad (\text{yoki } t = -\sqrt{a}x + \sqrt{ax^2+bx+c})$$

almashtirishni bajaramiz. U holda

$$ax^2+bx+c = t^2 - 2\sqrt{a}\,xt + ax^2,$$

$$x = \frac{t^2-c}{2\sqrt{a}t+b}, \qquad dx = \frac{2(\sqrt{a}t^2+bt+c\sqrt{a})}{(2\sqrt{a}t+b)^2}dt,$$

$$\sqrt{ax^2+bx+c} = \frac{\sqrt{a}t^2+bt+c\sqrt{a}}{2\sqrt{a}t+b}$$

bo‘ladi. Natijada

$$\int R(x, \sqrt{ax^2+bx+c})dx =$$

$$=\int R\left(\frac{t^2-c}{2\sqrt{a}t+b},\frac{\sqrt{a}t^2+bt+c\sqrt{a}}{2\sqrt{a}t+b}\right)\cdot\frac{2(\sqrt{a}t^2+bt+c\sqrt{a})}{(2\sqrt{a}t+b)^2}dt$$ bo‘ladi.

1-misol $\int \frac{dx}{x\sqrt{x^2-x-5}}=?$

$\sqrt{x^2-x-5}$ ifodada $a>0$ bo′lganligi uchun

$t+x=\sqrt{x^2-x-5}$ deb belgilash kiritamiz $t=\sqrt{x^2-x-5}-x$

$$t^2+2tx+x^2=x^2-x-5$$

$$t^2+5=-x-2tx$$

$$x=\frac{-t^2-5}{2t+1}$$

$$dx=\frac{(-2t)(2t+1)-2(-t^2-5)}{(2t+1)^2}dt$$

$$dx=\frac{-4t^2-2t+2t^2+10}{(2t+1)^2}dt$$

$$dx=\frac{-2t^2-2t+10}{(2t+1)^2}dt$$

x ning t ga bog′liq ifodasini dastlabki integralga olib borib qo′yamiz.

$$\int\frac{1}{\left(\frac{-5-t^2}{2t+1}\right)\left(t+\frac{-5-t^2}{2t+1}\right)}\frac{-2t^2-2t+10}{(2t+1)^2}dt$$

$$=\int\frac{(2t+1)^2}{(-5-t^2)(t^2+t-5)}\frac{-2t^2-2t+10}{(2t+1)^2}dt=$$

$$=\int\frac{-2}{(-5-t^2)}dt=\int\frac{2}{(5+t^2)}dt=\frac{2}{\sqrt{5}}arctg\left(\frac{t}{\sqrt{5}}\right)+c$$

$$=\frac{2}{\sqrt{5}}arctg\left(\frac{\sqrt{x^2-x-5}-x}{\sqrt{5}}\right)+c$$

2-holat. $c>0$ **bo‘lsin**. Bu holda (1) integralda ushbu

$$t=\frac{1}{x}(\sqrt{ax^2+bx+c}-\sqrt{c})$$

yoki

$$t=\frac{1}{x}\left(\sqrt{ax^2+bx+c}+\sqrt{c}\right)$$

almashtirishini bajaramiz. Unda

$$x=\frac{2\sqrt{c}\,t-b}{a-t^2}\quad,\qquad dx=\frac{\sqrt{c}t^2-bt+\sqrt{c}\,a}{(a+t)^2}dt\ ,$$

$$\sqrt{ax^2+bx+c}=\frac{\sqrt{c}\ t^2-bt+a\sqrt{c}}{a-t^2}$$

bo‘lib, (1) integral ratsional funksiyaning integraliga keladi:

$$\int R(x,\sqrt{ax^2+bx+c})dx=$$

$$=\int R\left(\frac{2\sqrt{c}\,t-b}{a-t^2}\ ,\ \frac{\sqrt{c}\,t^2-bt+a\sqrt{c}}{a-t^2}\right)\left(\frac{\sqrt{c}t^2-bt+\sqrt{c}\,a}{(a+t)^2}\right)dt$$

2-misol

$$\int\frac{dx}{x\sqrt{2+x-x^2}}=?$$

$\sqrt{2+x-x^2}\ ifodada\ c>0$ bo′lganligi uchun

$\sqrt{2+x-x^2}=tx-\sqrt{2}$ deb belgilash kiritamiz.

$$t=\frac{\sqrt{2+x-x^2}+\sqrt{2}}{x}$$

$$2+x-x^2=t^2x^2-2\sqrt{2}tx+2$$

$$1-x=t^2x-2\sqrt{2}t$$

$$x=\frac{2\sqrt{2}t+1}{t^2+1}$$

$$dx=\frac{2\sqrt{2}t^2+2\sqrt{2}-4\sqrt{2}t^2-2t}{(t^2+1)^2}dt$$

$$dx=\frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(t^2+1)^2}dt$$

x ning t ga bog′liq ifodasini dastlabki integralga olib borib qo′yamiz

$$\int\frac{1}{\left(\frac{2\sqrt{2}t+1}{t^2+1}\right)\left(t\left(\frac{2\sqrt{2}t+1}{t^2+1}\right)-\sqrt{2}\right)}\frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(t^2+1)^2}dt=$$

$$\int\frac{(t^2+1)^2}{(2\sqrt{2}t+1)(2\sqrt{2}t^2+t-\sqrt{2}t^2-\sqrt{2})}\frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(t^2+1)^2}dt=\int\frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(2\sqrt{2}t+1)(\sqrt{2}t^2+t-\sqrt{2})}dt=$$

$$\int\frac{-2(\sqrt{2}t^2-\sqrt{2}+t)}{(2\sqrt{2}t+1)(\sqrt{2}t^2+t-\sqrt{2})}dt=\int\frac{-2}{(2\sqrt{2}t+1)}dt=\frac{-2}{2\sqrt{2}}\ln\left(2\sqrt{2}t+1\right)+C=-\frac{\ln(2\sqrt{2}t+1)}{\sqrt{2}}+$$

$$+C=-\frac{\ln\left(2\sqrt{2}\left(\frac{\sqrt{2+x-x^2}+\sqrt{2}}{x}\right)+1\right)}{\sqrt{2}}+C$$

3-holat ax^2+bx+c kvadrat uchhad turli x_1 va x_2 haqiqiy ildizga ega bo‘lsin:

$ax^2+bx+c=a(x-x_1)\cdot(x-x_2)$.

Bu holda (1) integralda ushbu

$$t=\frac{1}{x-x_1}\sqrt{ax^2+bx+c}$$

almashtirishni bajaramiz. Natijada

$$x=\frac{-ax_2+x_1t^2}{t^2-a}\quad,\qquad \sqrt{ax^2+bx+c}=\frac{a(x_1-x_2)}{t^2-a}t$$

$$dx=\frac{2a(x_1-x_2)t}{(t^2-a)^2}dt$$ bo‘lib,

$$\int R(x,\sqrt{ax^2+bx+c})dx=$$

$$=\int R\left(\frac{-ax_2+x_1t^2}{t^2-a}\quad,\quad \frac{a(x_1-x_2)}{t^2-a}t\right)\cdot\frac{2a(x_1-x_2)t}{(t^2-a)^2}dt$$ bo‘ladi.

3-misol $\int\frac{dx}{(x-2)\sqrt{7x-x^2-10}}$

$\sqrt{7x-x^2-10}$ ifodada $D>0$ bo′lganligi uchun

$\sqrt{7x-x^2-10}=t(x-5)$ deb belgilash kiritamiz

$$t=\frac{\sqrt{7x-x^2-10}}{x-5}$$

$$7x-x^2-10=t^2(x-5)^2$$

$$(2-x)(x-5)=t^2(x-5)^2$$

$$2-x=t^2(x-5)$$

$$x=\frac{5t^2+2}{t^2+1}$$

$$dx=\frac{10t(t^2+1)-(5t^2+2)2t}{(t^2+1)^2}dt$$

$$dx=\frac{6t}{(t^2+1)^2}dt$$

x ning t ga bog′liq ifodasini dastlabki integralga olib borib qo′yamiz

$$\int\frac{1}{\left(\frac{5t^2+2}{t^2+1}-2\right)t\left(\frac{5t^2+2}{t^2+1}-5\right)}\frac{6t}{(t^2+1)^2}dt$$

$$=\int\frac{1}{\left(\frac{5t^2+2-2t^2-2}{t^2+1}\right)\left(\frac{5t^2+2-5t^2-5}{t^2+1}\right)t}\frac{6t}{(t^2+1)^2}dt=$$

$$\int \frac{(t^2+1)^2}{(3t^2)(-3)t}\frac{6t}{(t^2+1)^2}dt = \int \frac{-2}{3t^2}dt = \frac{2}{3}\left(\frac{1}{t}\right) + C = \frac{2}{3}\left(\frac{x-5}{\sqrt{7x-x^2-10}}\right) + C$$

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