

AYRIM IRRATIONAL KO'RINISHDAGI INTEGRALLARNI EYLER ALMASHTIRISHLARI YORDAMIDA RATSIONALLASHTIRISH

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Annotatsiya : $\int R(x, \sqrt{ax^2 + bx + c}) dx$ ko'rinishidagi integrallarni hisoblashda biz avvalo bu integral ostidagi ifodani ratsional funksiyaga keltirib olishimiz zarur. Buning uchun Eylarning 3 ta almashtirishi bizga yordam beradi. Quyida biz shu almashtirishlarni misollarda ko'rib chiqamiz.

Kalit so'zlar: Irratsional integral, ratsional funksiya, Eyler almashtirishlari, kvadrad uchhad.

Bizga irratsional ko'rinishdagi

$$\int R(x, \sqrt{ax^2 + bx + c}) dx \quad (1)$$

integral berilgan bo'lsin. Ushbu integralni quyidagi uchta almashtirish yordamida ratsional funksiya integraliga keladi. Odatda bu almashtirishlarni Eyler almashtirishlari deymiz. (1) integralda a, b, c -haqiqiy sonlar bo'lib, $ax^2 + bx + c$ kvadrat uchhad teng ildizlarga ega emas.

1-holat $a > 0$ bo'lsin.

(1) integralda ushbu

$$t = \sqrt{ax} + \sqrt{ax^2 + bx + c} \quad (\text{yoki } t = -\sqrt{ax} + \sqrt{ax^2 + bx + c})$$

almashtirishni bajaramiz. U holda

$$ax^2 + bx + c = t^2 - 2\sqrt{a}xt + ax^2,$$

$$x = \frac{t^2 - c}{2\sqrt{at} + b}, \quad dx = \frac{2(\sqrt{at}^2 + bt + c\sqrt{a})}{(2\sqrt{at} + b)^2} dt,$$

$$\sqrt{ax^2 + bx + c} = \frac{\sqrt{at}^2 + bt + c\sqrt{a}}{2\sqrt{at} + b}$$

bo'ladi. Natijada

$$\int R(x, \sqrt{ax^2 + bx + c}) dx =$$

$$= \int R\left(\frac{t^2 - c}{2\sqrt{at} + b}, \frac{\sqrt{a}t^2 + bt + c\sqrt{a}}{2\sqrt{a}t + b}\right) \cdot \frac{2(\sqrt{a}t^2 + bt + c\sqrt{a})}{(2\sqrt{a}t + b)^2} dt \quad \text{bo'ldi.}$$

1-misol $\int \frac{dx}{x\sqrt{x^2 - x - 5}} = ?$

$\sqrt{x^2 - x - 5}$ ifodada $a > 0$ bo'lganligi uchun

$$t + x = \sqrt{x^2 - x - 5} \text{ deb belgilash kiritamiz } t = \sqrt{x^2 - x - 5} - x$$

$$t^2 + 2tx + x^2 = x^2 - x - 5$$

$$t^2 + 5 = -x - 2tx$$

$$x = \frac{-t^2 - 5}{2t + 1}$$

$$dx = \frac{(-2t)(2t + 1) - 2(-t^2 - 5)}{(2t + 1)^2} dt$$

$$dx = \frac{-4t^2 - 2t + 2t^2 + 10}{(2t + 1)^2} dt$$

$$dx = \frac{-2t^2 - 2t + 10}{(2t + 1)^2} dt$$

x ning t ga bog'liq ifodasini dastlabki integralga olib borib qo'yamiz.

$$\int \frac{1}{\left(\frac{-5 - t^2}{2t + 1}\right)\left(t + \frac{-5 - t^2}{2t + 1}\right)} \frac{-2t^2 - 2t + 10}{(2t + 1)^2} dt$$

$$= \int \frac{(2t + 1)^2}{(-5 - t^2)(t^2 + t - 5)} \frac{-2t^2 - 2t + 10}{(2t + 1)^2} dt =$$

$$= \int \frac{-2}{(-5 - t^2)} dt = \int \frac{2}{(5 + t^2)} dt = \frac{2}{\sqrt{5}} \arctg\left(\frac{t}{\sqrt{5}}\right) + c$$

$$= \frac{2}{\sqrt{5}} \arctg\left(\frac{\sqrt{x^2 - x - 5} - x}{\sqrt{5}}\right) + c$$

2-holat. $c > 0$ bo'lsin. Bu holda (1) integralda ushbu

$$t = \frac{1}{x}(\sqrt{ax^2 + bx + c} - \sqrt{c})$$

yoki

$$t = \frac{1}{x}(\sqrt{ax^2 + bx + c} + \sqrt{c})$$

almashtirishini bajaramiz. Unda

$$x = \frac{2\sqrt{c}t - b}{a - t^2}, \quad dx = \frac{\sqrt{c}t^2 - bt + \sqrt{c}a}{(a + t)^2} dt,$$

$$\sqrt{ax^2 + bx + c} = \frac{\sqrt{c}t^2 - bt + a\sqrt{c}}{a - t^2}$$

bo'lib, (1) integral ratsional funksiyaning integraliga keladi:

$$\begin{aligned} & \int R(x, \sqrt{ax^2 + bx + c}) dx = \\ & = \int R\left(\frac{2\sqrt{c}t - b}{a - t^2}, \frac{\sqrt{c}t^2 - bt + a\sqrt{c}}{a - t^2}\right) \left(\frac{\sqrt{c}t^2 - bt + \sqrt{c}a}{(a + t)^2}\right) dt \end{aligned}$$

2-misol

$$\int \frac{dx}{x\sqrt{2+x-x^2}} = ?$$

$\sqrt{2+x-x^2}$ ifodada $c > 0$ bo'lganligi uchun

$\sqrt{2+x-x^2} = tx - \sqrt{2}$ deb belgilash kiritamiz.

$$t = \frac{\sqrt{2+x-x^2} + \sqrt{2}}{x}$$

$$2+x-x^2 = t^2x^2 - 2\sqrt{2}tx + 2$$

$$1-x = t^2x - 2\sqrt{2}t$$

$$x = \frac{2\sqrt{2}t + 1}{t^2 + 1}$$

$$dx = \frac{2\sqrt{2}t^2 + 2\sqrt{2} - 4\sqrt{2}t^2 - 2t}{(t^2 + 1)^2} dt$$

$$dx = \frac{-2\sqrt{2}t^2 + 2\sqrt{2} - 2t}{(t^2 + 1)^2} dt$$

x ning t ga bog'liq ifodasini dastlabki integralga olib borib qo'yamiz

$$\int \frac{1}{\left(\frac{2\sqrt{2}t+1}{t^2+1}\right)\left(t\left(\frac{2\sqrt{2}t+1}{t^2+1}\right)-\sqrt{2}\right)} \frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(t^2+1)^2} dt =$$

$$\int \frac{(t^2+1)^2}{(2\sqrt{2}t+1)(2\sqrt{2}t^2+t-\sqrt{2}t^2-\sqrt{2})} \frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(t^2+1)^2} dt = \int \frac{-2\sqrt{2}t^2+2\sqrt{2}-2t}{(2\sqrt{2}t+1)(\sqrt{2}t^2+t-\sqrt{2})} dt =$$

$$\int \frac{-2(\sqrt{2}t^2-\sqrt{2}+t)}{(2\sqrt{2}t+1)(\sqrt{2}t^2+t-\sqrt{2})} dt = \int \frac{-2}{(2\sqrt{2}t+1)} dt = \frac{-2}{2\sqrt{2}} \ln(2\sqrt{2}t + 1) + C = -\frac{\ln(2\sqrt{2}t+1)}{\sqrt{2}} +$$

$$+C = -\frac{\ln\left(2\sqrt{2}\left(\frac{\sqrt{2+x-x^2}+\sqrt{2}}{x}\right)+1\right)}{\sqrt{2}} + C$$

3-holat $ax^2 + bx + c$ kvadrat uchhad turli x_1 va x_2 haqiqiy ildizga ega bo'lsin:

$$ax^2 + bx + c = a(x - x_1) \cdot (x - x_2).$$

Bu holda (1) integralda ushbu

$$t = \frac{1}{x - x_1} \sqrt{ax^2 + bx + c}$$

almashtirishni bajaramiz. Natijada

$$x = \frac{-ax_2 + x_1 t^2}{t^2 - a}, \quad \sqrt{ax^2 + bx + c} = \frac{a(x_1 - x_2)}{t^2 - a} t$$

$$dx = \frac{2a(x_1 - x_2)t}{(t^2 - a)^2} dt \quad \text{bo'lib,}$$

$$\begin{aligned} & \int R(x, \sqrt{ax^2 + bx + c}) dx = \\ &= \int R\left(\frac{-ax_2 + x_1 t^2}{t^2 - a}, \frac{a(x_1 - x_2)}{t^2 - a} t\right) \cdot \frac{2a(x_1 - x_2)t}{(t^2 - a)^2} dt \quad \text{bo'ladi.} \end{aligned}$$

3-misol $\int \frac{dx}{(x-2)\sqrt{7x-x^2-10}}$

$\sqrt{7x - x^2 - 10}$ ifodada $D > 0$ bo'lganligi uchun

$$\sqrt{7x - x^2 - 10} = t(x - 5) \text{ deb belgilash kiritamiz}$$

$$t = \frac{\sqrt{7x - x^2 - 10}}{x - 5}$$

$$7x - x^2 - 10 = t^2(x - 5)^2$$

$$(2 - x)(x - 5) = t^2(x - 5)^2$$

$$2 - x = t^2(x - 5)$$

$$x = \frac{5t^2 + 2}{t^2 + 1}$$

$$dx = \frac{10t(t^2 + 1) - (5t^2 + 2)2t}{(t^2 + 1)^2} dt$$

$$dx = \frac{6t}{(t^2 + 1)^2} dt$$

x ning t ga bog'liq ifodasini dastlabki integralga olib borib qo'yamiz

$$\begin{aligned} & \int \frac{1}{\left(\frac{5t^2 + 2}{t^2 + 1} - 2\right) t \left(\frac{5t^2 + 2}{t^2 + 1} - 5\right)} \frac{6t}{(t^2 + 1)^2} dt \\ &= \int \frac{1}{\left(\frac{5t^2 + 2 - 2t^2 - 2}{t^2 + 1}\right) \left(\frac{5t^2 + 2 - 5t^2 - 5}{t^2 + 1}\right)} \frac{6t}{(t^2 + 1)^2} dt = \end{aligned}$$

$$\int \frac{(t^2 + 1)^2}{(3t^2)(-3)t} \frac{6t}{(t^2 + 1)^2} dt = \int \frac{-2}{3t^2} dt = \frac{2}{3} \left(\frac{1}{t} \right) + C = \frac{2}{3} \left(\frac{x - 5}{\sqrt{7x - x^2 - 10}} \right) + C$$

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