BIRMAN-SCHWINGER PRINCIPLE FOR TWO-PARTICLE DISCRETE SCHRÖDINGER OPERATORS ON LATTICES

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ABSTRACT

We consider a system of two identical bosons moving on the hypercubic lattice , Z^d, d ≥ 1 and interacting through attractive short range potentials μV (μ > 0) and the two-particle Schrödinger operators $H_\mu(k) = H_0$(k)-μV, kϵT^d associated to the system, where T^d is the d - dimensional torus. We study the Birman-Schwinger principle for two-particle Schrödinger operators.

Keywords: discrete Schrödinger operators, quantum mechanical two-particle, *Birman-Schwinger principle*, system, hamiltonians.

1.Introduction

We consider a system of two bosons moving on the d -dimensional hypercubic lattice Z^d interacting via short-range pair potentials. The main goal of this paper is to prove Birman-Schwinger principle for two-particle Schrödinger operators $H_\mu(k)$, k ∈T associated to the system of two identical bosons, which is not occur in the continuous case. The spectral properties for the two-particle lattice hamiltonians on the d -dimensional lattice Z^d, d ≥1.

2. **Birman-Schwinger principle for two-particle Schrödinger operator** $H_\mu(k)$, k ∈T. For a bounded self-adjoint operator A in a Hilbert space H, we define $n_+(\mu, A)$ resp. $n_-(\mu, A)$ as

$n_+(\gamma, A)$ = max{dim L: L⊂H ;(Af , f)>γ , f ∈ L,|| f ||= 1} resp.

$n_-(\gamma, A)$ = max{dim L: L⊂H ;(Af , f)<γ , f ∈ L,|| f ||= 1}

The value $n_+(\gamma, A)$ (resp. $n_-(\gamma, A)$) is equal to the infinity, if γ is in the essential spectrum and if $n_+(\gamma, A)$ (resp. $n_-(\gamma, A)$) is finite, it is equal to the number of the eigenvalues of A greater (resp. smaller) than γ (see Glazman lemma [14]).

Let d ≥1 and $\varepsilon_k(\cdot)$, k ∈ T be the two-particle dispersion relation and $\nu \epsilon l_0(Z^d; R_0^+)$ where $\nu \epsilon l_0(Z^d; R_0^+)$ is space of non-negative functions defined on Z^d and vanishing at infinity.

For any z<$\varepsilon_{min}(k)$ we define a non-negative compact Birman-Schwinger operator acting in $l^2(Z^d$ as

$$B_\mu(k,z) := V^{1/2} R_0(k,z) V^{1/2}$$

Where $R_0(k,z)$ zϵ $C/[\varepsilon_{min}(k), \varepsilon_{max}(k)]$, is the resolvent of the operator $H_0(k)$ and $V^{1/2}$ is the non-negative square root of the non-negative operator V :

($V^{1/2}\psi$)(x) = $v^{1/2}(x)\psi$)(x), $\psi\epsilon l^2(Z^d)$. The kernel function $B_\mu(k,z,\cdot,\cdot)$ kϵT^d z< $\varepsilon_{min}(k)$ of the Birman-Schwinger operator $B_\mu(k,z)$ is of the form

$$B_\mu(k,z,x,y) \coloneqq v^{1/2}(x) R_0(k,z,x-y) v^{1/2}(y) \;\; x,y\epsilon Z^d,$$

$$R_0(k,z,x) = \int_{T^d} \frac{\varepsilon^{i(p,x)}}{\varepsilon_k(p)-z} d\eta(p), \;\; x\epsilon Z^d.$$

In the following Lemma we formulate some important properties of the kernel function of the Birman-Schwinger operator.

Lemma 1.1 Let d ≥ 3. For any kϵT^d and $x,y\epsilon Z^d$ the function B(k,·; x, y) is realanalytic in zϵC \[($\varepsilon_{min}(k), \varepsilon_{max}(k)$].

Proof. For any kϵG and $x\epsilon Z^d$ the regularity of the function

$$R_0(k,z,x) = \int_{T^d} \frac{\varepsilon^{i(p,x)}}{\varepsilon_k(p)-z} d\eta(p), \;\; x\epsilon Z^d.$$

In zϵ $C/[\varepsilon_{min}(k), \varepsilon_{max}(k)]$ implies the regularity of the kernel B(k,·; x, y) in zϵ $C/[\varepsilon_{min}(k), \varepsilon_{max}(k)]$.

Lemma 1.2 (Birman-Schwinger principle). For any $< \varepsilon_{min}(k)$, kϵT^d the following assertions (i)--(iv) hold true.

1. If φ$\epsilon l^2(Z^d)$solves H_μ(k)φ = zφ , then ψ:=$V^{1/2}$φ$\epsilon l^2(Z^d)$ solves ψ = B(k,z)ψ .
2. If ψ$\epsilon l^2(Z^d)$ solves ψ = B(k,z)ψ , then φ:=$R_0(k,z)V^{1/2}$ψ$\epsilon l^2(Z^d)$ solves H_μ(k)φ= zφ .
3. z is an eigenvalue of H_μ(k) of multiplicity m if and only if 1 is an eigenvalue of B(k,z) of multiplicity m .
4. Counting multiplicities, the number N_-(z, H_μ (k)) − μ of eigenvalues of H_μ (k) less than z equals the number N_+(1,B(k,z) of eigenvalues of B(k,z) greater than 1, i.e., the equality

$$N_-(z, H_\mu(k)) = N_+(1, B(k,z))$$

holds.

Proof. Theorem 2.2 is the standard Birman-Schwinger principle for z<$\varepsilon_{min}(k)$ ( ) max z ε k and the proof for the lattice Schrödinger operator case can be found in [4].

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