CHIZIQSIZ REAKSIYA-DIFFUZIYALI EPIDEMIYA MASALASINI MATEMATIK MODELLASHTIRISH

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Masalaning qo‘yilishi. $Q=\{(t,x):\ 0\le t\le T,\ x\in R_+^1\}$ sohada reaksiya-diffuziya tenglamasi berilgan bo‘lsin[1]:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{\alpha}\frac{\partial u}{\partial x}\right)+k(x,t)u\left(1-u^{\beta}\right), \quad (1)$$

$$u|_{t=0}=u_0(x)\ge 0,\ x\in R_+^1,$$
$$u|_{x=0}=\varphi(t)\ge 0,\ 0\le t\le T. \quad (2)$$

(1)-(2) masalada $u_0(x)$ funksiya chekli va quyidagi shartlar bajariladigan funksiyalarni qaraymiz:

$$0<k(x,t)\le k(t)\in C(Q),\quad 0<\varphi(t)\in C^1(0,\infty)$$

$u(t,x)$ yechimni chiziqsiz ajratish usulidan foydalanib, quyidagi shaklda qidiramiz:

$$u(t,x)=\varphi(t)w(\tau(t),x) \quad (3)$$

$\varphi(t)$ quyidagi tenglamaning yechimi:

$$\frac{d\varphi}{dt}=k(t)\varphi\left(1-\varphi^{\beta}\right),\ \varphi(t)=\left(1+e^{-\beta\int_0^t k(y)dy}\right)^{-\frac{1}{\beta}} \quad (4).$$

Darhaqiqat, (4) va (3) ni hisobga olgan holda (1) ga invariant tenglamani hosil qilamiz:

$$\frac{\partial w}{\partial \tau}=\frac{\partial}{\partial x}\left(w^{\alpha}\frac{\partial w}{\partial x}\right)+\varphi_1(t)w\left(1-w^{\beta}\right),$$

bu yerda $\tau(t)$ yangi noma’lum funksiya va $w(\tau(t),x)$ aniqlanishi kerak bo‘lgan funksiya.

(3) ni (1) ga olib borib qo‘ysak, unda quyidagi tenglamaga kelamiz va bundan $\tau(t)$ ni tanlaymiz:

$$\frac{d\varphi}{dt}w+\varphi(t)\frac{\partial w}{\partial \tau}\frac{d\tau}{dt}=\varphi^{(\alpha+1)}\frac{\partial}{\partial x}\left(w^{\alpha}\frac{\partial w}{\partial x}\right)+k(t)\varphi(t)w\left(1-\varphi^{\beta}w^{\beta}\right),$$

$$k(t)\varphi\left(1-\varphi^{\beta}\right)w+\varphi(t)\frac{\partial w}{\partial \tau}\frac{d\tau}{dt}=\varphi^{(\alpha+1)}\frac{\partial}{\partial x}\left(w^{\alpha}\frac{\partial w}{\partial x}\right)+k(t)\varphi(t)w\left(1-\varphi^{\beta}w^{\beta}\right),$$

$\tau(t)$ ni $\frac{d\tau}{dt}\varphi^{-\alpha}=1$ shart bajariladigan qilib tanlaymiz va tenglamani olamiz:

$$\frac{\partial w}{\partial \tau}=\frac{\partial}{\partial x}\left(w^{\alpha}\frac{\partial w}{\partial x}\right)+k(t)\varphi^{\beta-\alpha}w\left(1-w^{\beta}\right), \qquad (5)$$

bu yerda $\frac{d\tau}{dt}=\varphi^{\alpha}$, $\varphi_1(t)=k(t)\varphi^{\beta-\alpha}$ va $\varphi(t)$ funksiya (4) da keltirilgan.

Shubhasiz, (4) ga binoan, agar $\int_0^t k(y)dy$ mavjud bo‘lsa, u holda $\lim_{t\to\infty}\varphi(t)=1$. Agar $\beta\neq\alpha$ bo‘lsa, biz t yetarlicha katta deb $\varphi_1(t)$ ~$k(t)$ taxmin qilishimiz mumkin. Bundan yana (1) ko‘rinishdagi tenglamani olamiz. Shu sababli $\varphi(t)$ funksiyani invariant deb ataymiz, bu yerda $u(t,x)=\varphi(t)w(\tau(t),x)$ (1) tenglamaning yechimi, $w(\tau(t),x)$ esa (5) tenglamaning yechimi va (5) tenglama (1) tenglamaning invariantidir.

1-teorema. Faraz qilaylik, $Q=\{(t,x): t>0,\, x\in R_+^1\}$ sohada $0\le u_0(x)\le 1,\ x\in R,\ \xi=\frac{|x|}{(T+\tau(t))^{\frac{1}{2+\alpha}}}$ o‘rinli bo‘lsin. U holda manfiy bo‘lmagan umumlashgan $u(t,x)$ yechim aniqlangan va yechim uchun quyidagi munosabat o‘rinli bo‘ladi.

$$\varphi(t)(T+\tau(t))^{-\frac{1}{2+\alpha}}\left(c-\frac{\alpha}{4}\xi^2\right)_+^{\frac{1}{\alpha}}\le u(t,x)\le e^{\int_0^t k(y)\varphi^{\beta-\alpha}(y)dy}(T+\tau(t))^{-\frac{1}{2+\alpha}}\left(c-\frac{\alpha}{4}\xi^2\right)_+^{\frac{1}{\alpha}}$$

bu yerda $\varphi(t)=\left(1+e^{-\beta\int_0^t k(y)dy}\right)^{-\frac{1}{\beta}}$, $(a)_+=\max(0,a)$.

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Teoremani isboti sifatida (1) masala uchun yechimlarni taqqoslash teoremasi bo‘yicha [2] yuqori bahoni va chiziqsiz ajratish usulini [1-2] qo‘llash orqali quyidan bahoni olamiz. Bundan yechimlarni solishtirish teoremasini [2] qo‘llash natijasida umumiy yechim uchun ushbu tenglikka ega bo‘lamiz:

$$u(t,x)=\varphi(t)(T+\tau(t))^{-\frac{1}{2+\alpha}}\left(c-\frac{\alpha}{4}\xi^2\right)_+^{\frac{1}{\alpha}},$$

Bu yerda $\tau(t)=\int_0^t e^{\int_0^z k(y)dy}dz$ va $T>0$.

Olingan yechim orqali chiziqsiz reaksiya-diffuziyali epidemiya masalasi uchun ayirmali sxema quramiz va sonli tahlilini amalga oshiramiz.

$Q=\{(t,x):\ 0\le t\le T,\ x\in R_+^1\}$ sohada quyidagi Koshi masalani uchun ayirmali sxema ko‘rib chiqaylik:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(u^{\alpha}\frac{\partial u}{\partial x}\right)+k(x,t)u(1-u^{\beta}), \quad (6)$$

$$u|_{t=0}=u_0(x)\ge 0,\ x\in R_+^1,$$
$$\begin{cases} u|_{x=x_0}=\varphi(t), & t\in[0,T], \\ u|_{x=x_n}=\varphi_1(t)=0, & t\in[0,T] \end{cases} \quad (7)$$

bu yerda, $0<k(x,t)\le k(t)\in C(Q),\ \ 0<\varphi(t)\in C^1(0,\infty)$.

$$\varphi(t)=\left(1+e^{-\beta\int_0^t k(y)dy}\right)^{-\frac{1}{\beta}},\ u_0(x,t)=\varphi(t)\left(c-\frac{\alpha}{4}\xi^2\right)_+^{\frac{1}{\alpha}},\ \xi=\frac{x}{(\tau(t))^{\frac{1}{2}}},\ \ \tau(t)=\int_0^t\varphi^{\alpha}(y)dy.$$

U holda boshlang‘ich va chegaraviy shartlar quyidagi ko‘rinishda bo‘ladi:

$$u(0,x)=\left(\frac{1}{2}\right)^{\frac{1}{\beta}}\left(c-\frac{\alpha}{4}\xi^2\right)_+^{\frac{1}{\alpha}},\ u(t,0)=\left(1+e^{-\beta\int_0^t k(y)dy}\right)^{-\frac{1}{\beta}},\ u(t,l)=0,\ l=2\sqrt{\frac{c}{\alpha}\tau(t)}.$$

(6), (7) masalaning sonli yechimi uchun $x\in R_+^1$ da x uchun h qadam bilan ϖ_h teng o‘lchovli

$$\varpi_h=\{x_i=d+ih,\ h>0,\ i=0,1,2,\dots,n,\ d+hn=b\}$$

va vaqt uchun

$$\varpi_{h_1}=\{t_j=jh_1,\ h_1>0,\ j=0,1,2,\dots,m,\ h_1m=T\}$$

bo‘lgan to‘r quramiz.

(9), (10) masala uchun balans usulini qo‘llagan holda quyidagi oshkormas ayirmali sxema bilan almashtiramiz va $O(h^2+h_1)$ hatolikka ega ayirmali masalani hosil qilamiz.

$$\begin{cases} \dfrac{u_{ij+1}-u_{ij}}{h_1}=\dfrac{1}{h^2}\left[a\left(u_{i+1j+1}\right)\left(u_{i+1j+1}-u_{ij+1}\right)-a\left(u_{ij+1}\right)\left(u_{ij+1}-u_{i-1j+1}\right)\right]+ \\ \quad +ku_{ij+1}\left(1-b\left(u_{ij+1}\right)\right), \qquad i=1,2,...,n-1; \quad j=0,1,...,m-1, \\ u_{i0}=u_0\left(x_i\right), \qquad i=0,1,...,n, \\ u_{0j}=\varphi\left(t_j\right), \qquad j=0,1,...,m, \\ u_{nj}=\varphi_1\left(t_j\right), \qquad j=0,1,...,m, \end{cases} \tag{8}$$

bu yerda $a(u_{ij})=\dfrac{u_{ij}^{\alpha}+u_{i-1j}^{\alpha}}{2}$ yoki $a(u_{ij})=\left(\dfrac{u_{ij}+u_{i-1j}}{2}\right)^{\alpha}$, $b(u_{ij+1})=(u_{ij})^{\beta}$ formulalar bilan hisoblanadi.

(8) algebraik tenglamalar sistemasi u_{ij+1} ga nisbatan chiziqsiz.

Chiziqsiz tenglamalar sistemasini yechish uchun har xil iteratsiya usullaridan foydalanamiz va quyidagini hosil qilamiz:

$$\begin{aligned} \frac{u_{ij+1}^{(s+1)}-u_{ij}^{(s+1)}}{h_1}=\frac{1}{h^2}\left[a\left(u_{i+1j}^{(s)}\right)\left(u_{i+1j+1}^{(s+1)}-u_{ij+1}^{(s+1)}\right)-a\left(u_{ij}^{(s)}\right)\left(u_{ij+1}^{(s+1)}-u_{i-1j+1}^{(s+1)}\right)\right]+ \\ +ku_{ij+1}^{(s)}\left(1-b\left(u_{ij+1}\right)\right), i=1,2,...,n-1; \quad j=0,1,...,m-1, \end{aligned} \tag{9}$$

bunda $s=0,1,2,...$.

1) $b\left(u_{ij+1}\right)=\left(u_{ij+1}^{(s)}\right)^{\beta}$ - Pikar usuli,

2) $b\left(u_{ij+1}\right)=u_{ij+1}^{(s+1)}\left(u_{ij+1}^{(s)}\right)^{\beta-1}$ - maxsus usul,

3) $b\left(u_{ij+1}\right)=\left(u_{ij+1}^{(s)}\right)^{\beta}+\beta\left(u_{ij+1}^{(s)}\right)^{\beta-1}\left(u_{ij+1}^{(s+1)}-u_{ij+1}^{(s)}\right)$ - Nyuton usuli.

Yuqorida berilgan usullardan birini qo‘llash orqali chiziqsizlikni yo‘qotish [3-4] mumkin va har bir usul uchun (10) belgilashlarni kiritamiz

1) Pikar usuli uchun: $A_i=\dfrac{h_1}{h^2}\cdot a(u_{ij+1}^{(s)})$, $B_i=\dfrac{h_1}{h^2}\cdot a(u_{i+1j+1}^{(s)})$, $C_i=A_i+B_i+1$, $F_i=u_{ij+1}^{(s+1)}+h_1\cdot k\cdot u_{ij+1}^{(s)}\cdot\left(1-\left(u_{ij+1}^{(s)}\right)^{\beta}\right)$, $s=0,1,2,...,$ $i=1,...,n-1$.

2) Maxsus usul uchun: $A_i=\dfrac{h_1}{h^2}\cdot a(u_{ij+1}^{(s)})$, $B_i=\dfrac{h_1}{h^2}\cdot a(u_{i+1j+1}^{(s)})$, $C_i=A_i+B_i+1+h_1\cdot k\cdot\left(u_{ij+1}^{(s)}\right)^{\beta}$, $F_i=u_{ij+1}^{(s+1)}+h_1\cdot k\cdot u_{ij+1}^{(s)}$, $s=0,1,2,...,$ $i=1,...,n-1$.

3) Nyuton usuli uchun:

$$A_i = \frac{h_1}{h^2}\cdot a(u_{ij+1}^{(s)}),\qquad B_i = \frac{h_1}{h^2}\cdot a(u_{i+1j+1}^{(s)}),\qquad C_i = A_i + B_i + 1 + h_1\cdot k\cdot\beta\cdot\left(u_{ij+1}^{(s)}\right)^{\beta},$$

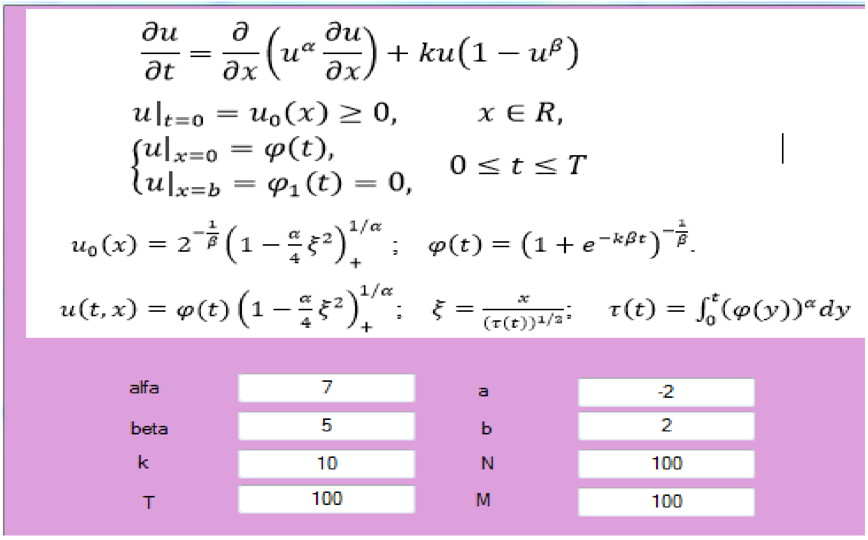
$$F_i = u_{ij+1}^{(s+1)} + h_1\cdot k\cdot u_{ij+1}^{(s)} + h_1\cdot k\cdot(\beta-1)\cdot\left(u_{ij+1}^{(s)}\right)^{\beta+1},\ s=0,1,2,...,\ i=1,...,n-1.$$

Ayirmali tenglamani quyidagi ko‘rinishda yozib olish mumkin:

$$A_i\cdot u_{i-1j+1}^{(s+1)} - C_i\cdot u_{ij+1}^{(s+1)} + B_i\cdot u_{i+1j+1}^{(s+1)} = -F_i,\ s=0,1,2,...,\ i=1,...,n-1,\ (11)$$

(11) tenglamalar sistemasini yechish uchun haydash (progonka) usulidan foydalaniladi va hamma sonli hisoblashlarda $\varepsilon = 10^{-3}$ deb qaraladi.

Qaralayotgan chiziqsiz reaksiya-diffuziyali epidemiya masalasini matematik modellashtirish uchun Visual Studio C# 2013 muhitida hamda MathCad 2001 packeti orqali dastur yaratildi va parametrlarning mos qiymatlarida quyidagi natijalar 1-2 rasmlarda berilgan:



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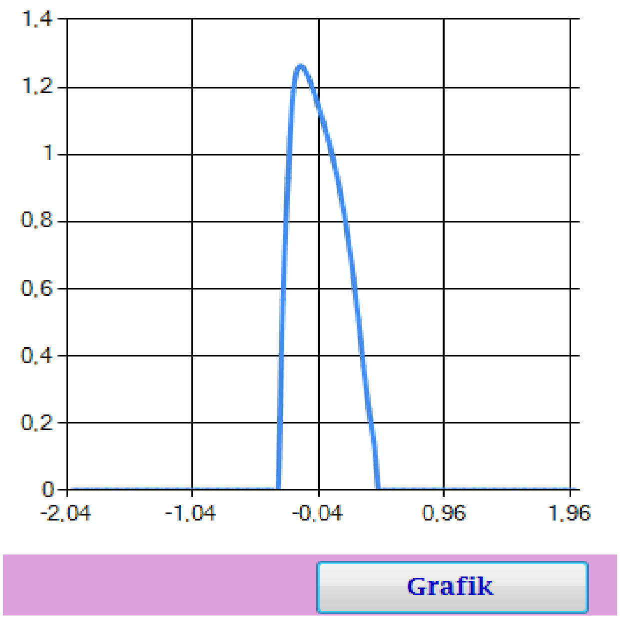
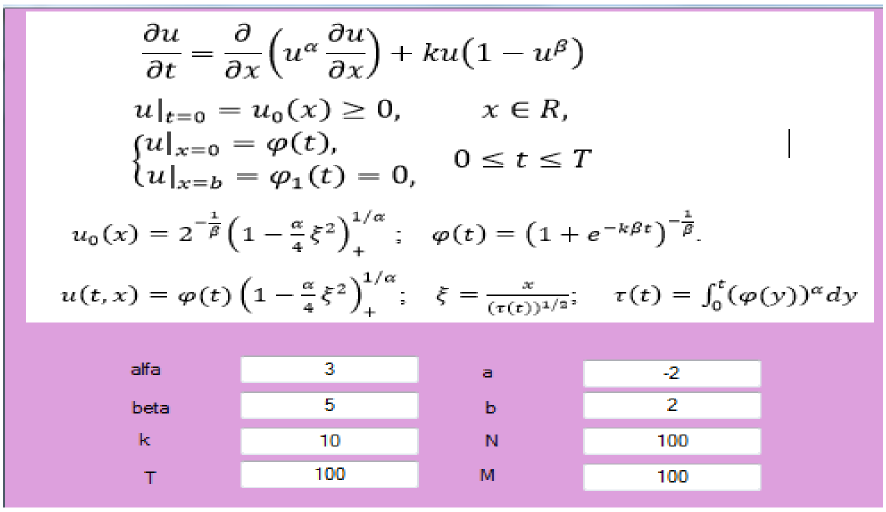
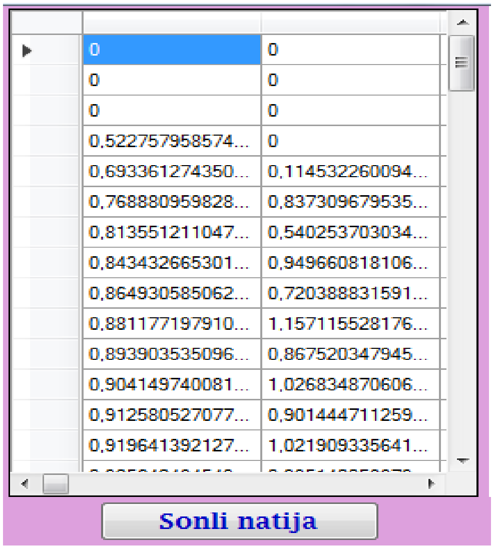
0,945281945637...	0,943746283154...
0,948153067200...	1,061181425107

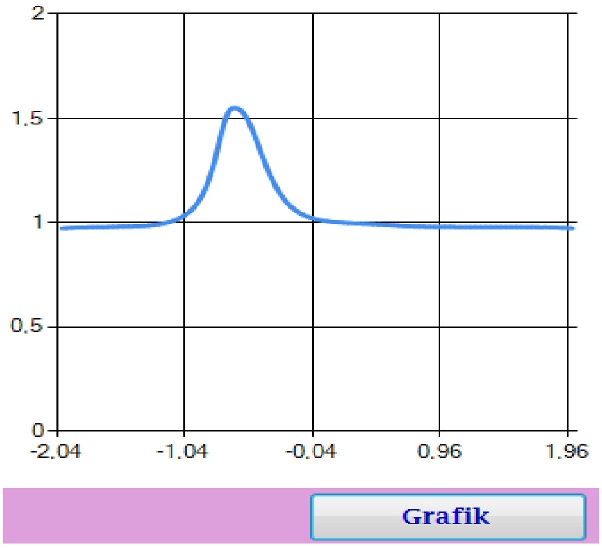
Sonli natija



1- 

rasm.





$\alpha = 7, \beta = 5, k = 10,$ hol uchun masalaning yechimi.

2-rasm. $\alpha = 3, \beta = 5, k = 10,$ hol uchun masalaning yechimi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI: (REFERENCES)

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