UYURMA-OQIM KO‘RINISHIDAGI NAVYE-STOKS TENGLAMALAR SISTEMASINI SONLI YECHISH

Xoliyorov Erkin Chorshanbiyevich
Termiz iqtisodiyot va servis universiteti
Iqtisodiyot va aniq fanlar kafedrasi dotsenti

Quyliyev Sardorbek Abdixoliqovich
Termiz davlat universiteti
Axborort texnologiyalari fakulteti,
Kampyuter tizimlari va uning dasturiy ta’minoti yo‘nalishi 2-kusr magistranti

ANNOTATSIYA

“Uyurma-oqim” ko‘rinishidagi Navye-Stoks tenglamalar sistemasini sonli modellashtirish masalasi qaralgan. Masalani sonli yechishda uyurma tenglamasi o‘zgaruvchan yo‘nalishli usullarga kiruvchi Pismen-Rekford sxemasi va yuqori relaksatsiyali iteratsiya usuli qo‘llaniladi. Aniq yechim bilan solishtirib yuqoridagi usullarning samarali ekanligi ko‘rsatildi.

KIRISH

Hozirgi vaqtda ikki o‘lchamli Navye-Stoks tenglamalarini sonli yechish uchun ko‘plab ishlar bag‘ishlangan. Bugungi kunga kelib, “uyurma-oqim” ko‘rinishidagi ikki o‘lchovli Navye-Stoks tenglamalari asosida yopishqoq siqilmaydigan suyuqlik muammolarini sonli modellashtirishga bag‘ishlangan ko‘plab tadqiqotlar mavjud.

Shunga qaramay, yuqoridagi muammoni sonli modellashtirish uchun ma’lum usullarni qo‘llash samaradorligi masalasi dolzarbdir.

[1] maqolada tabiiy o‘zgaruvchilarda Navye-Stoks tenglamalarini sonli yechish usuli taklif qilinadi. Usul harakat tenglamasi va uzluksizlik tenglamasini chekli ayirmali approksimatsiyasi yordamida birgalikda yechishga asoslangan. [2] da yopishqoq siqilmaydigan suyuqlikning (fizik o‘zgaruvchilarda) Navye-Stoks tenglamalarini yechishning sonli usuli berilgan. Taklif qilingan usulda, tenglamalar issiqlik o‘tkazuvchanligi tenglamalari bilan to‘ldirilgan. Uni qurishda dastlabki operatorlarni maxsus usulda fizik jarayonlarga bo‘lish bilan taqribiy faktorizatsiya sxemasidan foydalaniladi. [3] da tuzilgan to‘rlarda “tezlik-bosim” o‘zgaruvchilarida to‘liq Navye-Stoks tenglamalarini yechishda bosimni hisoblashning yangi yondashuvi taklif qilingan. Usul uzluksizlik tenglamasining integral shakllari va bosimning

parchalanishidan foydalanishga asoslangan bo‘lib, ular asosida yordamchi masala tuziladi.

[4] da, tezlik-bosim o‘zgaruvchilari Navye-Stoks tenglamalarining to‘liq sistemasi yopishqoq siqilmaydigan suyuqlik holati uchun sonli chekli ayirmalar usuli bilan yechiladi.

Navye-Stoks tenglamalarini “uyurma-oqim” ko‘rinishiga keltirish sonli modellashtirshni ancha osonlashtiradi. “Uyurma-oqim” ko‘rinishidagi Navye-Stoks tenglamalarini sonli modellashtirish bo‘yicha batafsil ma’lumotlarni [5, 6] da qarash mumkin.

Yuqoridagi ishlardan ayirmali o‘laroq, ushbu ishda “uyurma-oqim” Navye-Stoks tenglamalar sistemasini sonli yechish uchun o‘zgaruvchan yo‘nalishlar usullari (Pismen-Rekford sxemasidan) va yuqori relaksatsiyali iteratsiya usulini birgalikda qo‘llab sonli natijalar olingan. Bunday holda, (1) uyurma tenglamalarini sonli yechish uchun Pismen-Rekford usuli va (2) oqim funktsiyasi tenglamasini yechish uchun esa yuqori relaksatsiyali iteratsiya usuli yordamida sonli yechiladi.

1. Masalaning qo‘yilishi. Dekart koordinatalarida Navye-Stoks tenglamalari sistemasi «uyurma-oqim» ko‘rinishida quyidagicha yoziladi [7].

$$\frac{\partial\omega}{\partial t}+\frac{\partial\omega}{\partial x}\frac{\partial\psi}{\partial y}-\frac{\partial\omega}{\partial y}\frac{\partial\psi}{\partial x}=\nu\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)+Q(t,x,y), \quad (1)$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\omega, \quad (2)$$

$$\frac{\partial\psi}{\partial y}=u,\quad \frac{\partial\psi}{\partial x}=-v,\quad \omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \quad (3)$$

bu yerda x, y – fazoviy koordinatalar, t – vaqt u va v tezlik vektorining koordinata o‘qlaridagi proyeksiyasi, ν – kinematik qovushqoqlik koeffitsienti, ψ – oqim funksiyasi, ω – uyurma funksiyasi, Q – ma’lum funksiya.

$\overline{D}:\{(x,y,t)\in[0,1]\times[0,1]\times[0,T]\}$ sohada (1), (2) sistema uchun biz quyidagi chegaraviy shartlarni beramiz:

$$\psi|_{x=0}=0,\quad \psi|_{x=1}=0,\quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial\psi}{\partial x}\right|_{x=1}=0,\quad 0\le y\le 1, \quad (4)$$

$$\psi|_{y=0}=0,\quad \psi|_{y=1}=0,\quad \left.\frac{\partial\psi}{\partial y}\right|_{x=0}=0,\quad \left.\frac{\partial\psi}{\partial y}\right|_{x=1}=0,\quad 0\le x\le 1, \quad (5)$$

t=0 da boshlang‘*i*ch shartlar quy*i*dag*i* ko‘r*i*n*i*shga ega bo‘lad*i*:

$$\psi(0,x,y)=0,\quad \omega(0,x,y)=0. \qquad (6)$$

2. Chekli-ayirmali approksimatsiya. $\overline{D}$ sohada x, y fazoviy koordinatalarda tekis to‘rni kiritamiz:

$$\overline{\Omega}_h=\left\{x_i=ih,\ \ y_j=jh,\ \ 0\le i,j\le N,\ \ h=\frac{1}{N}\right\}$$

va t vaqt bo‘yicha to‘r ushbu ko‘rinishda kiritiladi:

$$\overline{\Omega}_\tau=\left\{t_k=k\tau,\ \ k=0,1,...M\right\},\text{ bunda } \tau=T/M.$$

(1), (2) differensial tenglamalar sistemasi $\overline{\Omega}=\overline{\Omega}_h\times\overline{\Omega}_\tau$ ayirmali to‘rda approksimatsiya qilinadi.

(1) uyurma tenglamasini sonli yechish uchun biz o‘zgaruvchan yo‘nalishlar usulini qo‘llaymiz (Pisman-Rekford sxemasi) [8]. Bu sxema oshkormas absolyut turg‘un sxemadir. Pismen-Rekford sxemasi approksimatsiya xatoligi $\theta(\tau^2+|h|^2)$, bu yerda $|h|^2=h_1^2+h_2^2$.

(1) tenglamani Pismen-Rekford sxemasi bilan approksimatisyalaymiz. Ushbu sxemada k qatlamdan $k+1$ qatlamga o‘tish ikki bosqichda amalga oshiriladi. Birinchi bosqichda oraliq qiymatlar $\omega_{i,j}^{k+1/2}$ quyidagi tenglamalar sistemasidan aniqlanadi

$$\frac{\omega_{i,j}^{k+1/2}-\omega_{i,j}^{k}}{0,5\tau}+\frac{\omega_{i+1,j}^{k+1/2}-\omega_{i-1,j}^{k+1/2}}{2h}\frac{\psi_{i,j+1}^{k}-\psi_{i,j-1}^{k}}{2h}-$$
$$-\frac{\omega_{i,j+1}^{k}-\omega_{i,j-1}^{k}}{2h}\frac{\psi_{i+1,j}^{k}-\psi_{i-1,j}^{k}}{2h}=\frac{\nu}{h^2}\left(\omega_{i+1,j}^{k+1/2}-2\omega_{i,j}^{k+1/2}+\omega_{i-1,j}^{k+1/2}\right)+$$
$$+\frac{\nu}{h^2}\left(\omega_{i,j+1}^{k}-2\omega_{i,j}^{k}+\omega_{i,j-1}^{k}\right)+Q\left(t_{k+1/2},x_i,y_i\right),$$
$$i,j=1,2,...,N-1\ \ k=0,1,...,M-1,$$

(7)

ikkinchi bosqichda esa topilgan $\omega_{i,j}^{k+1/2}$ qiymatlardan foydalanib, tenglamalar sistemasidagi $\omega_{i,j}^{k+1}$ qiymatlar quyidagi ayirmali tenglamalardan aniqlanadi

$$\frac{\omega_{i,j}^{k+1}-\omega_{i,j}^{k+1/2}}{0,5\tau}+\frac{\omega_{i+1,j}^{k+1/2}-\omega_{i-1,j}^{k+1/2}}{2h}\frac{\psi_{i,j+1}^{k}-\psi_{i,j-1}^{k}}{2h}-$$
$$-\frac{\omega_{i,j+1}^{k+1}-\omega_{i,j-1}^{k+1}}{2h}\frac{\psi_{i+1,j}^{k}-\psi_{i-1,j}^{k}}{2h}=\frac{\nu}{h^2}\left(\omega_{i+1,j}^{k+1/2}-2\omega_{i,j}^{k+1/2}+\omega_{i-1,j}^{k+1/2}\right)+$$
$$+\frac{\nu}{h^2}\left(\omega_{i,j+1}^{k+1}-2\omega_{i,j}^{k+1}+\omega_{i,j-1}^{k+1}\right)+Q(t_{k+1},x_i,y_i),$$
$$i,j=1,2,...,N-1\;\;k=0,1,...,M-1.$$
(8)

Oqim funksiyasi uchun (2) tenglama quyidagi ayirma sxemasi bilan approksimatsiyalanadi

$$\frac{\psi_{i+1,j}^{k+1}-2\psi_{i,j}^{k+1}+\psi_{i-1,j}^{k+1}}{h^2}+\frac{\psi_{i,j+1}^{k+1}-2\psi_{i,j}^{k+1}+\psi_{i,j-1}^{k+1}}{h^2}=-\omega_{ij}^{k+1}$$

yoki

$$\frac{\psi_{i+1,j}^{k+1}+\psi_{i-1,j}^{k+1}+\psi_{i,j+1}^{k+1}+\psi_{i,j-1}^{k+1}-4\psi_{i,j}^{k+1}}{h^2}=-\omega_{ij}^{k+1} \quad (9)$$
$$i,j=1,2,...,N-1\;\;k=0,1,...,M-1.$$

(3) tenglama quyidagicha approksimatsiya qilinadi

$$u_{i,j}^{k+1}=\frac{\psi_{i,j+1}^{k+1}-\psi_{i,j-1}^{k+1}}{2h},\quad v_{i,j}^{k+1}=-\frac{\psi_{i+1,j}^{k+1}-\psi_{i-1,j}^{k+1}}{2h} \quad (10)$$
$$i,j=1,2,...,N-1\;\;k=0,1,...,M-1.$$

3. Sonli yechish algoritmi. Keling, oddiy iteratsiya usulini qo‘llash uchun (9) ayirmali tenglamalarni quyidagi qulay shaklda qayta yozamiz [9]

$$\psi_{ij}^{k+1,s+1}=\psi_{ij}^{k+1,s}+r_{ij}, \quad (11)$$

Bu yerda

$$r_{ij}=\frac{\psi_{i+1,j}^{k+1,s}+\psi_{i-1,j}^{k+1,s}+\psi_{i,j+1}^{k+1,s}+\psi_{i,j-1}^{k+1,s}-4\psi_{i,j}^{k+1,s}+h^2w_{ij}^{k+1}}{4}.$$
(12)

Iteratsiya yaqinlashuvi tezligi yuqori relaksatsiyali iteratsiya usulini qo‘llash bilan yaxshilanadi [10]. Ketma -ket yuqori relaksatsiyali iteratsiya usuli quyidagi iteratsiya sxemadan foydalanadi

$$\psi_{ij}^{k+1,s+1}=\psi_{ij}^{k+1,s}+\vartheta\left(\frac{\psi_{i+1,j}^{k+1,s}+\psi_{i-1,j}^{k+1,s}+\psi_{i,j+1}^{k+1,s}+\psi_{i,j-1}^{k+1,s}-4\psi_{i,j}^{k+1,s}+h^2w_{ij}^{k+1}}{4}\right)=$$
$$=\psi_{ij}^{k+1,s}+\vartheta r_{ij},$$
(13)

bu erda parametr ϑ hududga tegishli $1 \le \vartheta < 2$. ϑ parametrni optimal tanlash chiziqli sistemalar uchun iteratsiya matritsalarning xos qiymatlariga bog‘liq va ko‘rib chiqilayotgan masala uchun quyidagi formula bo‘yicha beriladi. [4].

$$\vartheta = \frac{4}{2+\sqrt{4-4\cos^2\frac{\pi}{N-1}}} = \frac{4}{2+2\sqrt{1-\cos^2\frac{\pi}{N-1}}} = \frac{2}{1+\sin\frac{\pi}{N-1}}.$$

Oqim funksiyasi uchun ayirma tenglamalarning **Ошибка! Источник ссылки не найден.** oddiy iteratsiya, iteratsiyalar soni bo‘yicha yechimlari $n_0(\varepsilon)$ talab qilinadi. Belgilangan aniqlikka erishish uchun [5] ε aniqlikda formula bilan aniqlanadi .

$$n_0(\varepsilon) \approx \frac{2\ln(1/\varepsilon)}{\pi^2 h^2}, \qquad (14)$$

Yuqori relaksatsiya usuli uchun mos keladigan iteratsiyalar soni quyidagi shaklga ega:

$$n_0(\varepsilon) \approx \frac{2\ln(1/\varepsilon)}{\pi h}. \qquad (15)$$

$h = 0,05$, $\varepsilon = 10^{-3}$ bo‘lganda (14) va (15) formulalar bo‘yicha talab qilinadigan maksimal iteratsiyalar soni taxminan 560 va 88 ga teng bo‘ladi. Ko‘rinib turibdiki, oqim tenglamasini yechish uchun yuqori relaksatsiya usulidan foydalanish maqsadga muvofiqdir .

(7),(8) ayirmali tenglamalarni yechish algoritmini batafsil bayon etamiz.

Dastlab (7) tenglama ushbu standart shaklga keltiriladi

$$\bar{A}_i \omega_{i-1,j}^{k+1/2} - \bar{C}_i \omega_{i,j}^{k+1/2} + \bar{B}_i \omega_{i+1,j}^{k+1/2} = -\bar{F}_{i,j}^{k},$$
$$i,j = 1,2,\dots,N-1 \quad k = 0,1,\dots,M-1. \qquad (16)$$

Bu yerda

$$\bar{A}_i = 0,5\tau\left[\frac{\nu}{h^2} - \frac{1}{4h}\left(\psi_{i,j+1}^{k} - \psi_{i,j-1}^{k}\right)\right], \quad \bar{B}_i = 0,5\tau\left[\frac{\nu}{h^2} + \frac{1}{4h}\left(\psi_{i,j+1}^{k} - \psi_{i,j-1}^{k}\right)\right],$$

$$\bar{C}_i = 1 + \frac{\tau\nu}{h^2}, \qquad \bar{F}_{i,j}^{k} = \omega_{i,j}^{k} + \frac{0,5\tau\nu}{h^2}\left(\omega_{i,j+1}^{k} - 2\omega_{i,j}^{k} + \omega_{i,j-1}^{k}\right) +$$
$$+\frac{0,5\tau}{4h}\left(\omega_{i,j+1}^{k} - \omega_{i,j-1}^{k}\right)\left(\psi_{i+1,j}^{k} - \psi_{i-1,j}^{k}\right) + Q\left(t_{k+1/2}, x_i, y_i\right).$$

(16) ayirmali tenglama progonka usuli bilan yechiladi , buning uchun ayirmali to‘rning barcha tugunlaridagi $\omega_{ij}^{k+1/2}$ qiymatlarni aniqlash uchun $O\left(N^2\right)$ arifmetik amallar kerak bo‘ladi.

Barcha $\omega_{ij}^{k+1/2}$ lar topilgandan so‘ng, (8) ayirmali tenglamani yechish uchun uni standart shaklga keltiramiz:

$$A_j\omega_{i,j-1}^{k+1} - C_j\omega_{i,j}^{k+1} + B_j\omega_{i,j+1}^{k+1} = -F_{i,j}^{k+1/2},\quad i,j = 1,2,\dots,N-1\ \ k = 0,1,\dots,M-1. \tag{17}$$

Bu yerda

$$A_j = 0.5\tau\left[\frac{\nu}{h^2} - \frac{1}{4h}\left(\psi_{i,j+1}^{k} - \psi_{i,j-1}^{k}\right)\right],\quad B_j = 0.5\tau\left[\frac{\nu}{h^2} + \frac{1}{4h}\left(\psi_{i,j+1}^{k} - \psi_{i,j-1}^{k}\right)\right],$$

$$C_j = 1 + \frac{\tau\nu}{h^2},\quad F_{i,j}^{k+1/2} = \omega_{i,j}^{k+1/2} + \frac{0,5\tau\nu}{h^2}\left(\omega_{i+1,j}^{k+1/2} - 2\omega_{i,j}^{k+1/2} + \omega_{i-1,j}^{k+1/2}\right) +$$

$$+\frac{0,5\tau}{4h}\left(\omega_{i+1,j}^{k+1/2} - \omega_{i-1,j}^{k+1/2}\right)\left(\psi_{i,j+1}^{k} - \psi_{i,j-1}^{k}\right) + Q\left(t_{k+1}, x_i, y_j\right).$$

(17) ayirmali tenglamani progonka usuli bilan ω_{ij}^{k+1} larni topish uchun $O\left(N^2\right)$ arifmetik amallar talab qilinadi . Taqqoslash uchun shuni ta’kidlaymizki, uyurma tenglamasining ikki o‘lchovli oshkormas sxemasini Gauss usuli yordamida yechish uchun $O\left(N^6\right)$ arifmetik amallarni talab qilanadi.

(13), (16), (17) tenglamalar quyidagi chegaraviy shartlari bilan to‘ldiriladi

$$\psi_{i,0}^{k+1} = 0,\quad \psi_{i,N}^{k+1} = 0,\quad i = 0,1,\dots,N\ \ k = 0,1,\dots,M-1. \tag{18}$$

$$\psi_{0,j}^{k+1} = 0,\quad \psi_{N,j}^{k+1} = 0,\quad j = 0,1,\dots,N\ \ k = 0,1,\dots,M-1.. \tag{19}$$

va chegaraviy tugunlarda uyurma qiymatlarini aniqlash uchun ushbu Vuds shartlari [6] qo‘llaniladi :

$$\omega_{i,0}^{k+1} + \frac{\omega_{i,1}^{k+1}}{2} = \frac{3\left(\psi_{i,0}^{k} - \psi_{i,1}^{k}\right)}{h^2},\qquad \omega_{i,N}^{k+1} + \frac{\omega_{i,N-1}^{k+1}}{2} = \frac{3\left(\psi_{i,N}^{k} - \psi_{i,N-1}^{k}\right)}{h^2}, \tag{20}$$

$$\omega_{0,j}^{k+1} + \frac{\omega_{1,j}^{k+1}}{2} = \frac{3\left(\psi_{0,j}^{k} - \psi_{1,j}^{k}\right)}{h^2},\qquad \omega_{N,j}^{k+1} + \frac{\omega_{N-1,j}^{k+1}}{2} = \frac{3\left(\psi_{N,j}^{k} - \psi_{N-1,j}^{k}\right)}{h^2}. \tag{21}$$

(16),(20) va (17),(21) ayirmali tenglamalar sistemalari progonka usuli bilan yechiladi .

(16),(20) chegaraviy masalalar progonka usuli bilan yechish algoritmini keltiramiz:

$$\omega_{i,j}^{k+1/2} = \bar{\alpha}_{i+1}\omega_{i+1,j}^{k+1/2} + \bar{\beta}_{i+1}, \ .$$
$$i = N-1, N-2, ..., 1, 0, \ (0 < j < N), \ k = 0,1,...,M-1$$
(22)

$$\bar{\alpha}_{i+1} = \frac{\bar{B}_i}{\bar{C}_i - \bar{A}_i\bar{\alpha}_i}, \quad \bar{\beta}_{i+1} = \frac{\bar{A}_i\bar{\beta}_i + \bar{F}_{ij}^k}{\bar{C}_i - \bar{A}_i\bar{\alpha}_i},$$
$$i = 1, 2, ..., N-1, \ (0 < j < N), \ k = 0,1,...,M-1.$$
(23)

$$\omega_{0,j}^{k+1/2} = \bar{\alpha}_1\omega_{1,j}^{k+1/2} + \bar{\beta}_1, \qquad \omega_{0,j}^{k+1/2} = -\frac{1}{2}\omega_{1,j}^{k+1/2} + \frac{3\left(\psi_{0j}^k - \psi_{1j}^k\right)}{h^2},$$
$$\bar{\alpha}_1 = -0,5, \quad \bar{\beta}_1 = \frac{3\left(\psi_{0j}^k - \psi_{1j}^k\right)}{h^2}, \quad \left(0 < j < N\right),$$
(24)

$$\omega_{N,j}^{k+1/2} + \frac{\omega_{N-1,j}^{k+1/2}}{2} = \frac{3\left(\psi_{N,j}^k - \psi_{N-1,j}^k\right)}{h^2}, \qquad (25)$$
$$\omega_{N-1,j}^{k+1/2} = \bar{\alpha}_N\omega_{N,j}^{k+1/2} + \bar{\beta}_N, \qquad (26)$$

(26) ni (25) ga qo‘yib to‘rning $i = N$ chegaraviy tugunidagi qiymatni aniqlash uchun ushbu ifodaga ega bo‘lamiz:

$$\omega_{N,j}^{k+1/2} = \left[\frac{3\left(\psi_{N,j}^k - \psi_{N-1,j}^k\right)}{h^2} - 0,5\bar{\beta}_N\right] \Big/ \left(1 + 0,5\bar{\alpha}_N\right),$$
(27)

endi esa (17), (21) chegaraviy masalani progonka usuli bilan yechish algoritmini keltiramiz:

$$\omega_{i,j}^{k+1} = \alpha_{j+1}\omega_{i,j+1}^{k+1} + \beta_{j+1}, \quad j = N-1, N-2, ..., 1, 0, \quad (0 < i < N),$$
(28)

$$\alpha_{j+1} = \frac{B_j}{C_j - A_j\alpha_j}, \quad \beta_{j+1} = \frac{A_j\beta_j + F_{i,j}^{k+1/2}}{C_j - A_j\alpha_j}, \quad j = 1, 2, ..., N-1, \ (0 < i < N),$$
(29)

$$\omega_{i,0}^{k+1} = \alpha_1\omega_{i,1}^{k+1} + \beta_1, \quad \omega_{i,0}^{k+1} = -0,5\omega_{i,0}^{k+1} + 3\left(\psi_{i,0}^k - \psi_{i,1}^k\right)/h^2,$$
$$\alpha_1 = -0,5, \quad \beta_1 = 3\left(\psi_{i,0}^k - \psi_{i,1}^k\right)/h^2, \quad \left(0 < i < N\right),$$
(30)

$$\omega_{i,N}^{k+1}+0,5\omega_{i,N-1}^{k+1}=3\left(\psi_{i,N}^{k}-\psi_{i,N-1}^{k}\right)\Big/h^{2},\qquad(31)$$

$$\omega_{i,N-1}^{k+1}=\alpha_{N}\omega_{i,N}^{k+1}+\beta_{N},\qquad(32)$$

shundan so‘ng, yuqoridagiga o‘xshash $j=N$ da $\omega_{i,N}^{k+1}$ quyidagi ko‘rinishda aniqlanadi:

$$\omega_{i,N}^{k+1}=\left[\frac{3\left(\psi_{i,N}^{k}-\psi_{i,N-1}^{k}\right)}{h^{2}}-0,5\beta_{N}\right]\Big/\left(1+0,5\alpha_{N}\right).$$

(33)

3. Sonli natijalari va xulosalar. Yuqoridagi usullar asosida Navye-Stoks tenglamasini yechish uchun sonli hisoblar natijalarini keltiramiz .

(7)-(10) masalani sonli yechish uchun Pismen-Rekford va yuqori relaksatsiyali iteratsiya usullari qo‘llandi. To‘rlar quyidagicha tanlandi:. $h_x=h_y=0.05,\ \nu=1,\ \tau=0.001$. Sinov funktsiyasi usuli yordamida hisoblash tajribasini o‘tkazamiz. Agar differensial masala $\psi(t,x,y)=t\cdot\sin^{2}\pi x\cdot\sin^{2}2\pi y$ aniq yechimga ega bo‘lsa, u holda $\omega(t,x,y)$ va $Q(t,x,y)$ uchun ifodani aniqlashimiz mumkin[7].

Joriy (2) tenglamadan biz funktsiya uchun formulalarni $\omega(x,y)$ olamiz :

$$\omega(t,x,y)=-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}.\qquad(34)$$

Keling, hosilalarni topaylik

$$\frac{\partial^2\psi}{\partial x^2}=2\pi^2 t\cos 2\pi x\sin^2 2\pi y,\qquad \frac{\partial^2\psi}{\partial y^2}=$$

$$8\pi^2 t\cos 4\pi y\sin^2 2\pi x$$

Topilgan hosilalarni berib , **Ошибка! Источник ссылки не найден.** funktsiya uchun quyidagi formulalarni $\omega(t,x,y)$ olamiz :

$$\omega(t,x,y)=-2\pi^2 t(-10\cdot\sin^2\pi x\cdot\sin^2 2\pi y+4\cdot\sin^2\pi x+\sin^2 2\pi y)$$

uyurma tenglamasidan (1) biz uchun $Q(t,x,y)$

$$Q(t,x,y)=\frac{\partial\omega}{\partial t}+\frac{\partial\omega}{\partial x}\cdot\frac{\partial\psi}{\partial y}-\frac{\partial\omega}{\partial y}\cdot\frac{\partial\psi}{\partial x}-\nu\left(\frac{\partial^2\omega}{\partial x^2}-\frac{\partial^2\omega}{\partial y^2}\right)\qquad(35)$$

(34) ni (35) ga qo‘yib $Q(t,x,y)$ uchun quyidagi ifodaga ega bo‘lamiz:

$$Q(t,x,y)=-\left(\frac{\partial^3\psi}{\partial x^2\partial t}+\frac{\partial^3\psi}{\partial y^2\partial t}\right)-\left(\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^3\psi}{\partial x\partial y^2}\right)\frac{\partial\psi}{\partial y}+$$
$$+\left(\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right)\frac{\partial\psi}{\partial x}+\nu\left(\frac{\partial^4\psi}{\partial x^4}+2\frac{\partial^4\psi}{\partial x^2\partial y^2}+\frac{\partial^4\psi}{\partial y^4}\right),\qquad(36)$$

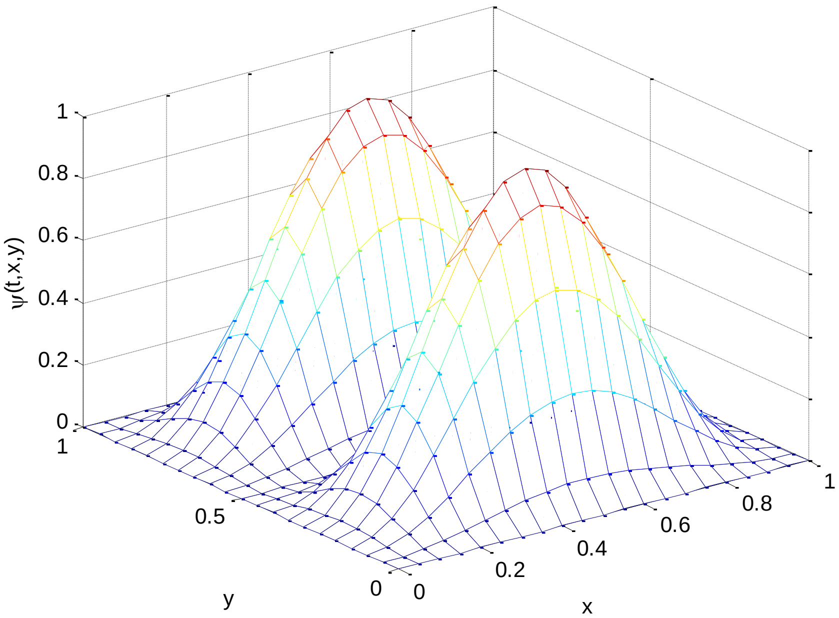
$\psi(t,x,y)$ funksiyasi xususiy hosilalarini (36) ga qo‘yib $Q(t,x,y)$ uchun quyidagi ifodaga ega bo‘lamiz:

$$Q(t,x,y) = 2\pi^2 \cdot (-10\sigma_1\sigma_2^2 + 4\sigma_1 + \sigma_2^2) +$$
$$+8\pi^4 t \cdot (44\sigma_1\sigma_2^2 - 22\sigma_1 - 76\sigma_2^2 + 4) +$$
$$+16\pi^4 t \cdot \cos\pi x \cdot \sigma_3. \sin\pi x \cdot \sigma_2^3/(10 \cdot \cos^2\pi x - 9) -$$
$$-32\pi^4 t^2 \cdot \cos\pi x \cdot \sigma_3 \sin^3\pi x \cdot \ \sigma_2 \cdot (5\sigma_3^2 - 3),$$

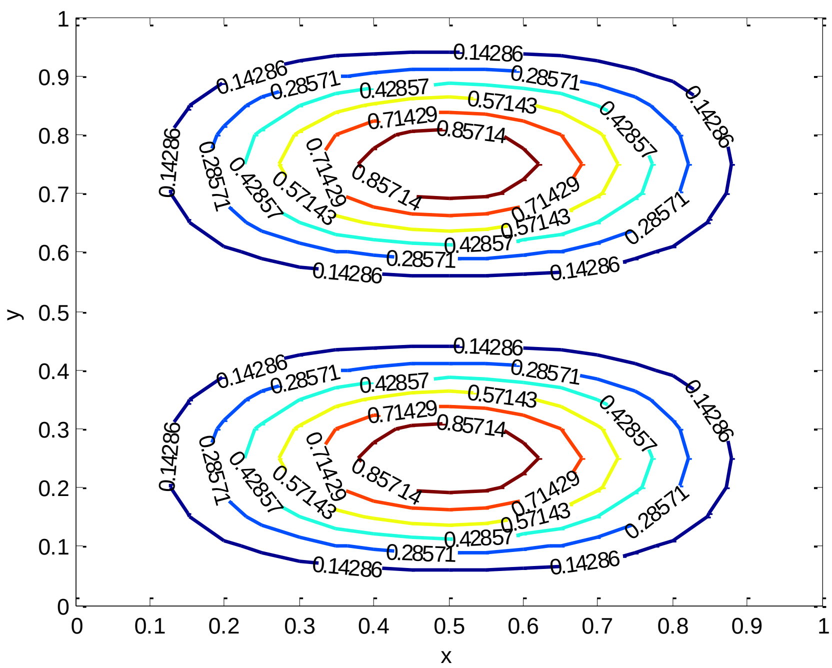
bu yerda $\sigma_1 = \sin^2\pi x$, $\sigma_2 = \sin 2\pi y$ $\sigma_3 = \cos 2\pi y$.

1-rasm.$\psi(T,x,y)$ o qim funktsiyalari analitik yechimi

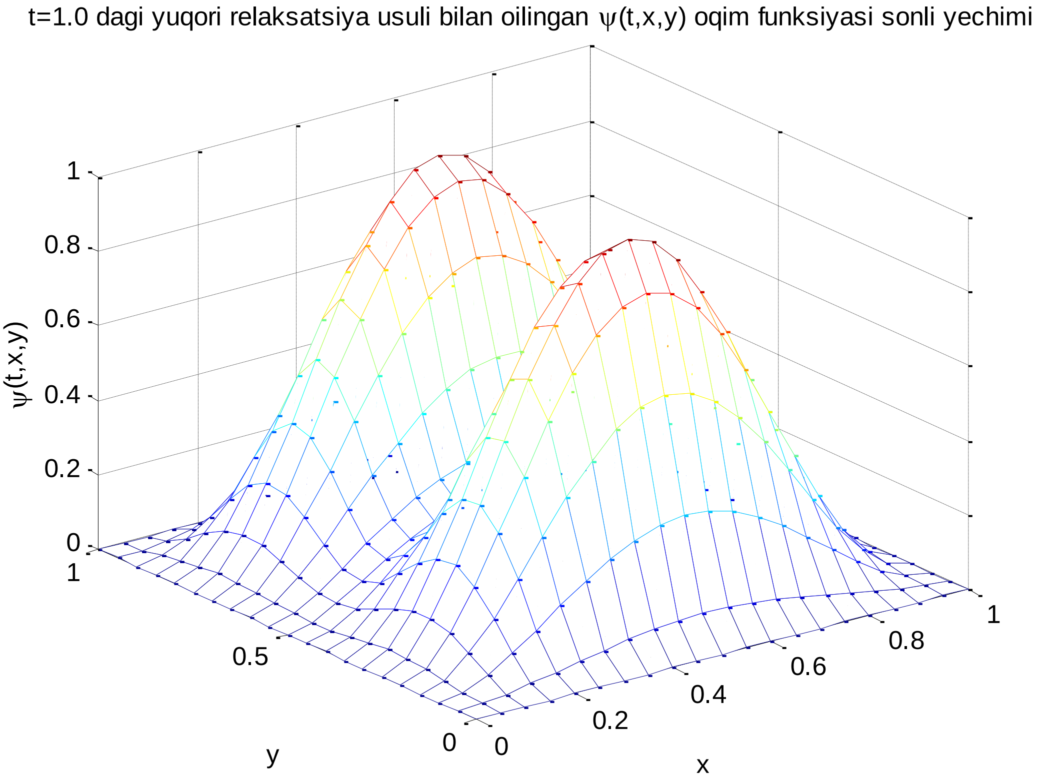
a



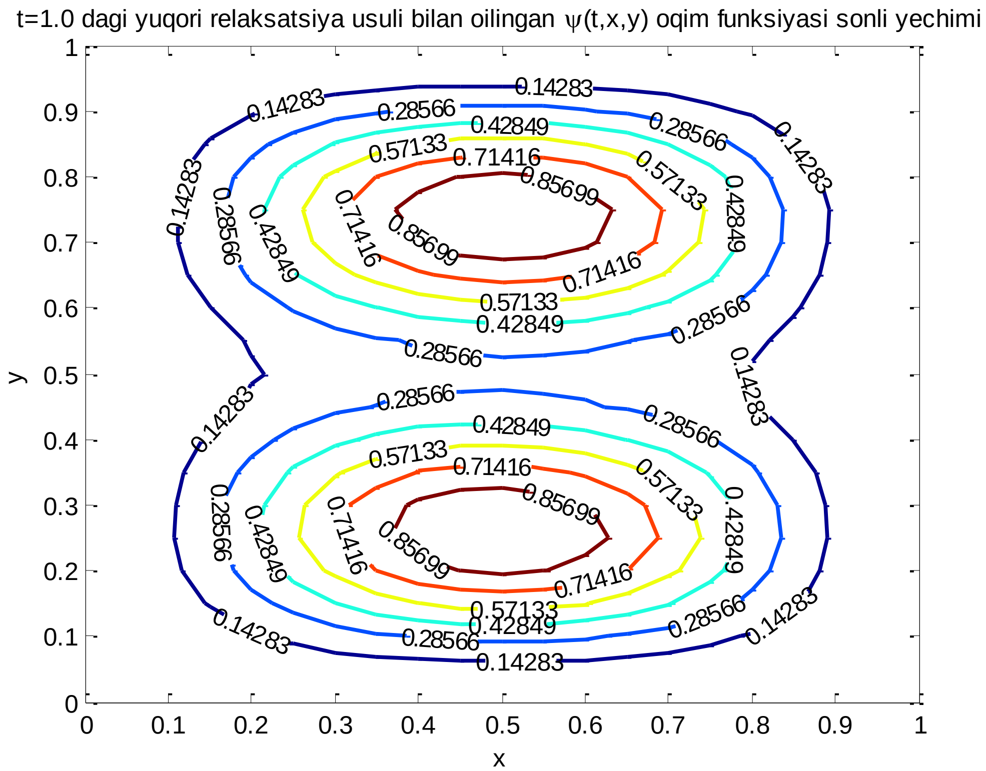
b



a

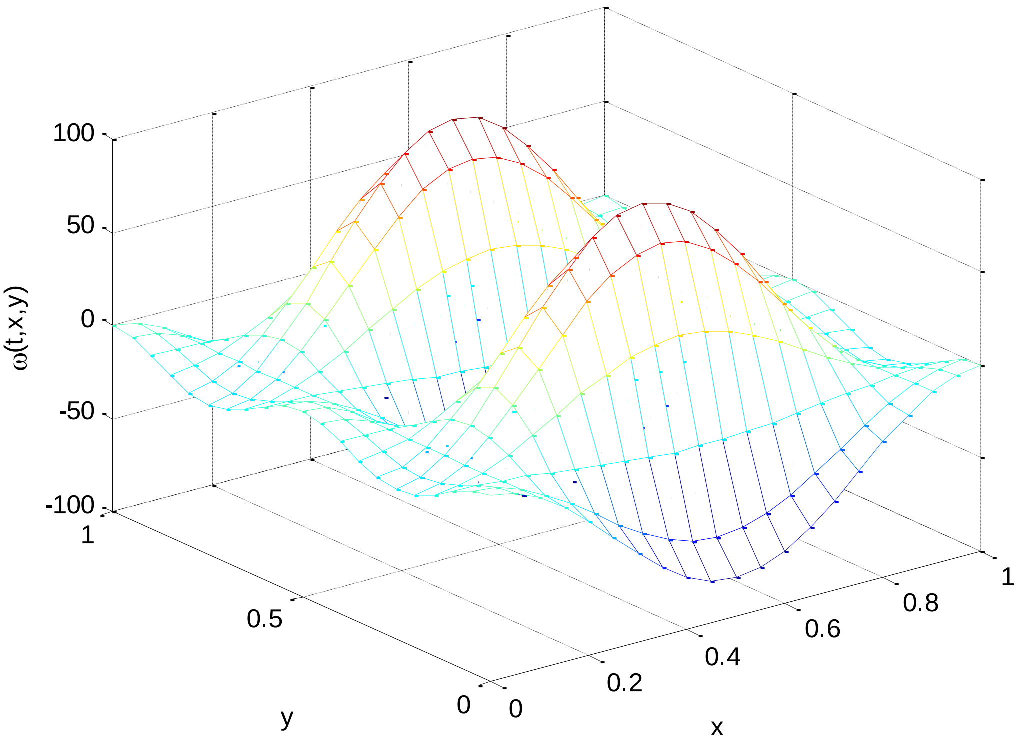

t=1.0 dagi yuqori relaksatsiya usuli bilan oilingan ψ(t,x,y) oqim funksiyasi sonli yechimi
ψ(t,x,y)
y
x

b

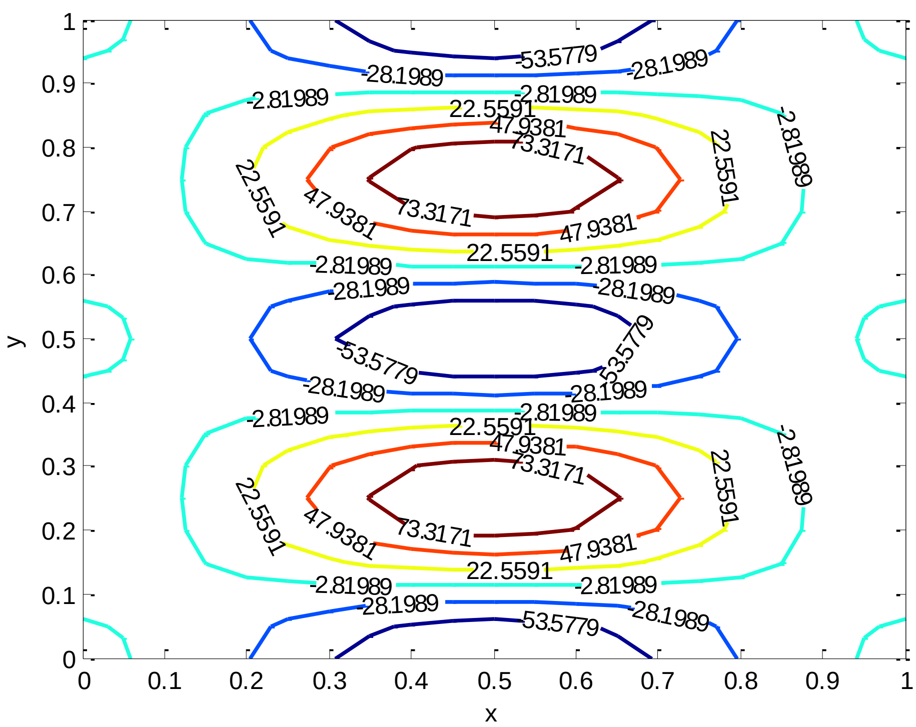

t=1.0 dagi yuqori relaksatsiya usuli bilan oilingan ψ(t,x,y) oqim funksiyasi sonli yechimi
y
x
0.14283
0.28566
0.42849
0.57133
0.71416
0.85699

2-rasm.$\psi(T,x,y)$ o qim funktsiyalari sonli yechimi

a

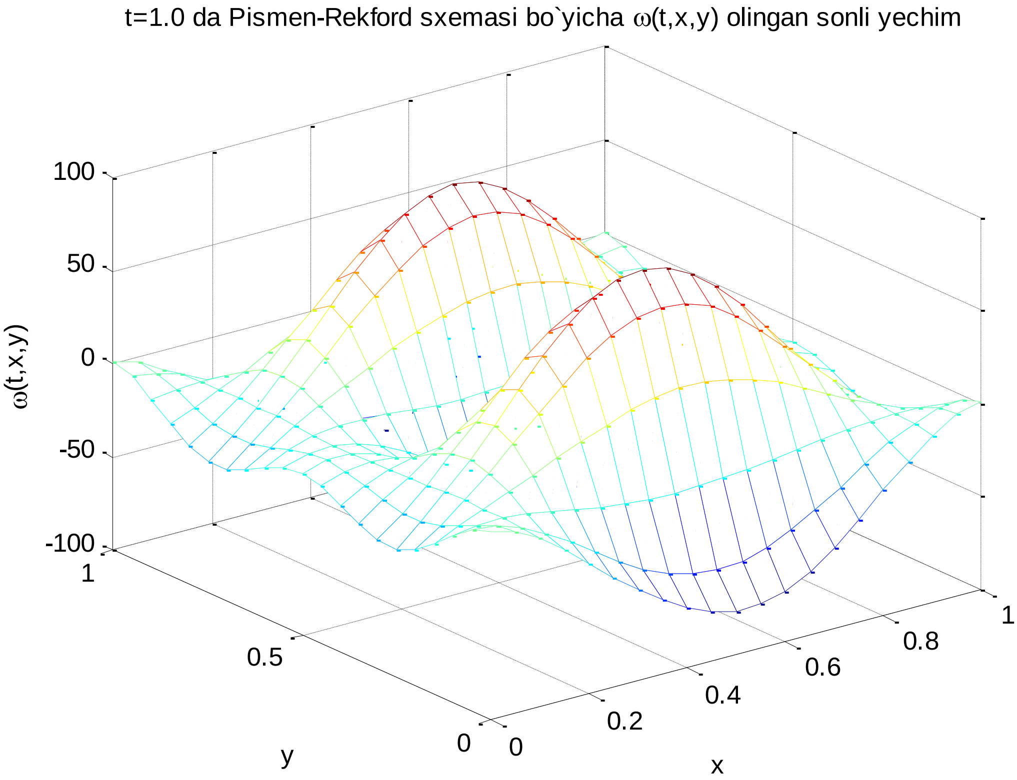


b

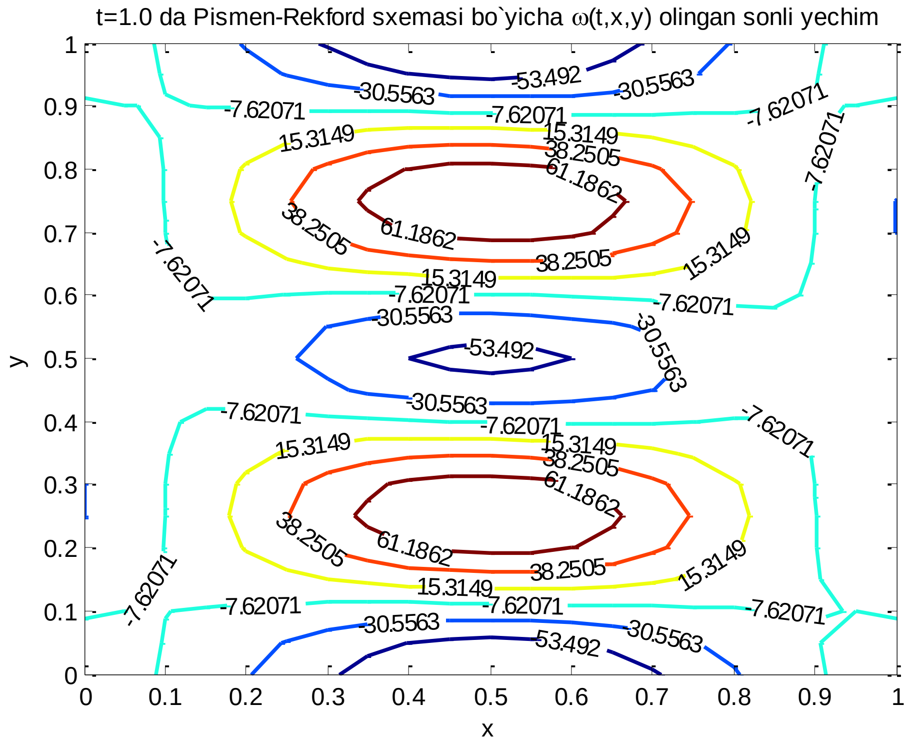


3-rasm. $\omega(T, x, y)$ uyurma funktsiyalari analitik yechimi

a



b



4-rasm.$\omega(T,x,y)$ uyurma funktsiyalari sonli yechimi

Mos ravishda 1-va 3-asmlarda t=1 da $\psi(t,x,y)$ oqim va $\omega(t,x,y)$ uyurmalarning analitik yechimi sirt va sath chizziqlari ko‘rinishida tasvirlangan.

2-va 4-rasmlarda esa t=1 da $\psi(t,x,y)$ oqim tenglamasining va $\omega(t,x,y)$ uyurma tenglamasining sonli yechimlari tasvirlangan.

1- va 4- rasmlardagi natijalardan ko‘rinadiki, aniq(analitik) va sonli yechimlar deyerli ustma-ust tushmoqda. Bundan “uyurma - oqim” ko‘rinishidagi Navye-Stoks tenglamalar sistemasini sonli yechish uchun Pismen – Rekford usuli va yuqori relaksatsiya iteratsiya usullarini birgalikda qo‘llash yaxshi natija berar ekan.

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