OPTIMAL DISCRETE CONTROL SYNTHESIS RESEARCH

Rustamova Malika Bakhodirovna
Senior Lecturer
Karshi branch Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi
malixonbonu@mail.ru

ABSTRACT

The article deals with the issues of synthesis of a discrete control system for multidimensional dynamic objects using the methods of graph theory and calculus of variations.

As a criterion for the quality of the control system, minimization of the total squares of the deviation of the values of the variables from the given one was taken. The calculation of transient processes is carried out with a hybrid use of topological and interpolation methods. Accounting for the delay property of the control objects is provided by discarding zero elements from the generated matrix of coefficients. The optimal control, formed for the synthesis of the control device, is defined as a function of the deviation of the output variables from the reference and previous control actions. This representation of the functioning of the control device allows you to fully take into account the dynamics of the controlled system in the development of control actions and is distinguished by the convenience of implementation in the microcontroller. The described control synthesis approach is convenient in that it is enough to have an object model in the form of a transition function, which is relatively easy to obtain as a result of active or pseudo-active experiments on the object, or as a result of statistical processing of data collected in a passive way. The analysis of the obtained results showed the adequacy of the solutions, but the latter is distinguished by convenience and speed.

Keywords: control, optimization dynamic system synthesis, algorithm, method calculus of variations, modeling method.

1. INTRODUCTION

The presence of microprocessor means in the control loop requires the development of a highly efficient method and algorithm for controller synthesis for optimal control of complex multidimensional dynamic systems.

The paper proposes a method and algorithms for the synthesis of discrete controls for continuous linear dynamic objects based on the topological interpolation approach and the calculus of variations. [1.2].

During the development, the goal was to obtain such a form of a microprocessor controller in order, on the one hand, to take into account the dynamics of the controlled system and, on the other hand, to eliminate the need to include the so-called “observer” in the controller, which restores unobservable variables using the model. In addition, the goal was to develop the most algorithmic method and eliminate the need to enter any formulas in the process of synthesizing a particular controller. [3-5].

2. Solution Method. There is a dynamic system with m_1 entrances and m_2 outputs described by the vector difference equation

$$\vec{X}_{k+1} = C \cdot \vec{X}_k + D \cdot \vec{U}_k$$

$$F = \sum_{j_1=1}^{k-1} \left(\sum_{i=1}^{m_2} q_1 \sum_{n=1}^{N} (Y_{m_1}^{зад} - Y_{m_1})^2 + \sum_{j=1}^{m_1} r_1 \, U_{jj_1}^2 \right) \quad (2)$$

where $i = \overline{1, m_1}$ - entry number; j=$\overline{1,(k-1)}$ - moment of impact; N - number of counted points on the control period; k - number of control intervals $T_и$ on the time of the transient process of the system;

$U_{ij} = U_i(t_0 + (j-1)T)$- control effect on i - th input: t_0 - the initial moment for which the state of the

where $\vec{X}_k$ - system state vector at time k, the dimension of the vector is determined by the structure and system complexity; $\vec{U}_k$ - control vector, dimension m; C ва D - communication matrices. Control actions are fed to the input of the system with a control period T_u. It is required to find such a sequence of optimal control actions $\vec{U}$, in order to minimize the total squared deviations of output processes over a certain time interval $\vec{Y}$ from assignment Y^{tas}. As an optimization interval, we take the time of the transient process of the dynamic system. The criterion will include all the squared deviations of the output variables, taken with their own weights at moments that are multiples of the control period T_u. Considering that in the general case, this value can be relatively large, for better optimization, we will include in the criterion and the squared deviations of the output variables on intermediate N time intervals at moments, multiples of T_u / N.

The output processes in the system are functions of control actions. The condition for optimal control for minimizing criterion (2) is the fulfillment of the requirements:

$$\frac{\partial F}{\partial U_{ij}} = 0 \quad (3)$$

$$\frac{\partial^2 F}{\partial U_{ij}^2} > 0, \; i = \overline{1, m_1}, \; j=\overline{1,(k-1)} \quad (4)$$

subject to the constraints in the form of linear equations (1).

To solve the problem, we use the values of the transition function obtained on the basis of the topological interpolation method [6] of each transmission channel:

$$Y_{m_1} = \sum_{j_2=1}^{m_1} \sum_{i_1=1}^{j_1} h^{ij_2}\ U_{(ji)_2} \quad (5)$$

$i = \overline{1,m_2}$- exit number; $i_2 = j_1 - i_1 + 1$ - moment of time; $j_2 -$ exit number; $h_{i_3}^{ij_2} = h^{ij}(t_{i3})$ the value of the transition function over the transmission channel from j_2 entrance i exit at time $i_3 = N(i_1 - 1) + n,\ t_{i_3} = i_3 \cdot T / N.$

The second partial derivatives (4) will always be positive, which is ensured by the positive values of the weights $q_i,\ i = \overline{1,m_2}$ и r_j, и $j = \overline{1,m_1}$. Substituting equation (5) into expression (2), we perform the operations of partial differentiation of the criterion F according to the desired U (3). After reducing like terms, we get the system $m_1(k-1)$ equations with $m_1(k-1)$ unknown.

In matrix form

$$A \cdot U = B, \quad (6)$$

where U - the vector of all dimension controls $m_1(k-1)$, A - matrix of the left side of the system of equations, the elements of which are obtained by the formula:

$$a_{i_3 j_3} = \sum_{i=1}^{m_2} q_i \sum_{j=j_4}^{k-n_1} \sum_{n_2=1}^{N} h_j^{ij_2} * h_{j_0}^{ij_1} \quad n = \overline{1,(k-1)},\ n_1 = \overline{1,n}\ ,\ j_1 = \overline{1,m_1}\ :$$

$$\begin{cases} \overline{1,m_1}\ ,\text{при}\ \ n \neq n_1\ , \\ \overline{1,J_1}\ ,\quad \text{при}\ n = n_1\ , \end{cases}$$

$$i_3 = m_1(n_1 - 1) + j_2\ , i_1 = m_1(n_1 - 1) + j_1 \quad (7)$$

$$i_4 = 1 + n - n_1; \qquad j_5 = N(j-1) + n_2;\ j_6 = N(j - 1 + n_1 - 1)n_2;$$

$$a_{ij} = a_{ji}\ ; \quad i = \overline{1,(m_1(k-1)-1)}; \quad j = \overline{1,(i-1)}$$

$$a_{ij} = a_{ji} + r_j; \quad i = m_1(i_1 - 1) + j; \quad j = \overline{1,m_1}; \quad i_1 = \overline{1,(k-1)}$$

In the formation of a symmetrical matrix A discrete values of the transition functions of the object are involved h_l^{ij}, multiples received at the time $\frac{T_u}{N}$, $l = \overline{1,N(k-1)}$ channel transmission from j - th entrance, $j = \overline{1,m_1}$ before i - exit, $i = \overline{1,m_2}$. The object's transmission channels may have delays greater than $\frac{T_u}{N}$, then the first values of the transition functions will be equal to zero. The value will decrease accordingly. K. If the system is in zero initial conditions, then the values of the output variables must be corrected by the desired controls by the value of the task

$Y_i^{зад}$, $i = \overline{1,m_2}$ [8].

If the system has a non-zero state, then the elements of the right side of the system of equations are determined in accordance with the functions $Y_{i_1}^{кор}$ for which you need to adjust i-th free motion output process $Y_{ij}^{св}$ on the optimized time interval $[t_0, t_0 + j \cdot T_u / N]$, to get given output processes $Y_{ij}^{зад}$.

$$Y_{ij} = Y_{ij}^{зад} = Y_{ij}^{св}, \quad i = \overline{1, m_2}\ , j = \overline{1, N(k-1)} \qquad (8)$$

Output processes are given by discrete values with a step $T_и/N$ $Y_{ij}^{зад} = Y_{ij}^{зад}(t_0 + j * \frac{T_и}{N})$. Free processes $Y_{ij}^{зад} = Y_{ij}^{св}(t_0 + j * \frac{T_и}{N})$ are defined using bipartite dynamic graph models.

We express the output processes $Y_{ij}^{св}$ free movement of the system with self-alignment through the current values of the output variables at the first moment of time Y_{ij} at every exit $i = \overline{1, m_2}$

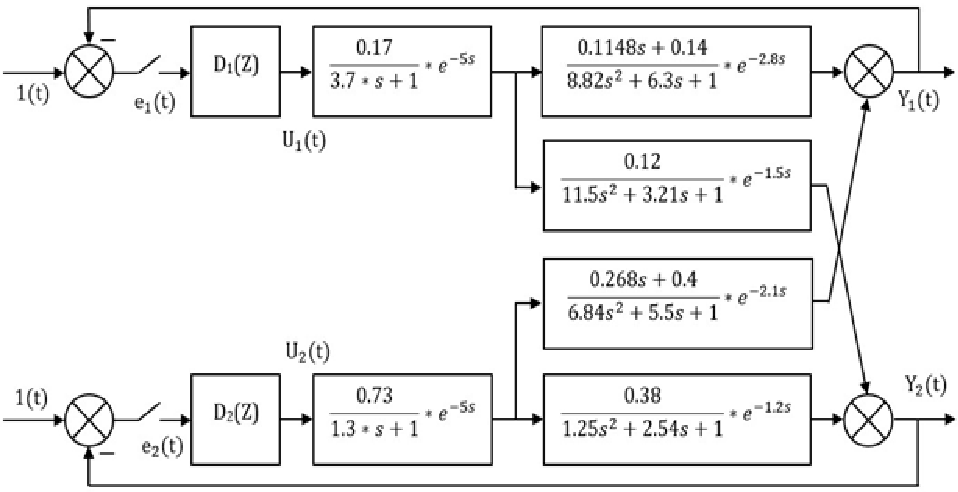
$$Y_{ij}^{св} = Y_{ij} + \sum_{n_1=1}^{m_1} \sum_{n=1}^{2} (h_{j-1+n}^{m_1} - h_n^{m_1}) U^{П} \qquad (9)$$

where $U_{n_1 n}^{П}$ - memorized by the controller control action on n_1 exit point in time $(-nT_u)$ relative to the current time of the moment of development of optimal controls; z - number of control intervals per transient time $z = \text{whole}(t_{nn} / T_u)$. [5].

Let us define the right side of the system of equations (9), which is formed for the synthesis of the controller that generates the optimal control as a function of the deviations of the output variables from the reference and the previous control actions applied to the controlled system at a given time interval

$$\overline{U_k^0} = E * \overline{e_k} + \sum_{i=1}^{z} G_j * \overline{U_{k-1}} \qquad (10)$$

This form of the controller makes it possible to fully take into account the dynamics of the controlled system when developing optimal controls and, at the same time, is very convenient for the implementation micromachine.



Fugure. 1. Structural diagram of the digital control system

$$B_{i3} = \sum_{i=1}^{m_2} \left(q_i \sum_{j=1+\Delta k}^{k-n_1+\Delta k} \sum_{n_{2=1}}^{N} h_{j_5}^{ij_2} (Y_{ij_6}^{зад} - Y_{i_1}) \right) - \sum_{n_3=1}^{z} \sum_{j_1=1}^{m_1} (\sum_{i=1}^{m_2} q \sum_{j=1+\Delta k}^{k-n_1+\Delta k} h_{j_5}^{ij_2} (h_{n_5}^{ij_2} - h_{n_4}^{ij_1}) \cdot U_{j_1 n_3}^{П}); \quad n_1 = \overline{1, (k-1)};$$

$$j_2 = \overline{1, m_1};\ i_3 = m_1(n_1 - 1) + j_2; \quad j_5 = N(j-1) + n_2; \quad n_4 = N \cdot n_3;$$
$$j_6 = N(j + n_1 - 2 + n_3) + n_1 + 1;$$
$$n_5 = N(j + n_1 - 2 + n_3) + n_2 \qquad (11)$$

Controls satisfying the system thus obtained from $m_1(k-1)$ linear equations are optimal in the sense of minimizing the criterion (2). Need to find the first m_1 unknowns that match the controls $U_{ij},\ i = \overline{1, m_1}$. To do this, by sequential elimination, starting with the last one, we get rid of all, except for the first, variables in the system of equations. Then, moving in the reverse order, we pass to the system with the first two unknowns, substitute the found expression for the first input control into one of the equations and find the second one. And so on until all are known. m_1 input controls for the first moment of timeTo facilitate the solution of this problem, we propose the following form of representation of the vector matrix B [7].

Taking into account expressions (4-5) to obtain the controller in forte (10), we obtain the equations for obtaining matrix elements B [11].

$$B(i,j) = \sum_{s=1}^{k-i+1} Yd_{s+1} h_{d^2,s};$$
$$k_1 = k(d-1); \quad k_2 = k(m_5 - 1); \quad m_4 = |j - i + k_1 - k_2|; \quad d = \overline{1, m_1}$$
$$m_5 = h_9 + 1; \quad m_3 = d + m_1 h_9; \quad h_9 = (j-1)/k;$$
$$i = \overline{1, k \cdot m_1}; \quad j = \overline{1, k \cdot m_1}$$
$$A(i,j) = \begin{cases} i = k_1 = j - k_2; & \sum\limits_{s=1}^{m_5 k - j + 1} h_{3,s} h_{d^2,s} \\ i - k_1 > j - k_2 \quad u \quad j > k_2; & \sum\limits_{s=1}^{k-i+1+k_1} h_{d^2,s} h_{m_3, s+m_4} \\ & \sum\limits_{s=1}^{m_5 k - j + 1} h_{m_3,s} h_{d^2, s+m_4} \end{cases}$$

The analysis of the obtained results showed the adequacy of the solutions, but the latter is distinguished by convenience and speed. The proposed method is also applicable to one-dimensional objects. In this case, the problem statement has the following formulation: it is required to determine piecewise constant control actions U supplied to the input of the object with a constant specified period T, to minimize the sum quadratic performance criterion:

$$F = \sum_{s=1}^{k} (Y_i - Y^{зад})^2 \to \min \qquad (12)$$

where $U_i = U((i-1)T)$; sought discrete values of control step actions; $Y_i = Y((i-1)T)$ discrete values of the controlled variable of the object; $Y^{зад}$ - set value of the output variable; k - the number of control periods on the optimization interval, it is enough to take no more than the time of the transition process of the control object t_m.

Consider processes with zero initial states of the object. In this case, the output variable is determined by the formula

$$Y_i = \sum_{j=1}^{i-1} h_i U_{i-j} \qquad (13)$$

where $h_j = h(jT)$ - moment of the transition function of the object. Let's solve the problem by the least squares method. On the optimization interval, we have $(k-1)$ unknown values of control actions U_i. Accordingly, differentiating the quality functional F for each of the unknowns, we obtain a system of equations:

$$\frac{dF}{dU_i} = 0, \quad i = 1, (k-1) \quad (14)$$

In this case, the condition:

$$\frac{d^2F}{dU_i^2} > 0, \; i = 1, (k-1) \quad (15)$$

Substituting the equation (14) в (15)

$$F = (h_1 U - Y^{зад})^2 + (h_1 U_2 + h_2 U_1 - Y^{зад})^2 + ...$$

$$+ (\sum_{j=1}^{k-1} h_j U_{k-j} - Y^{зад})^2 \; (16)$$

Performing the operations of differentiating the criterion

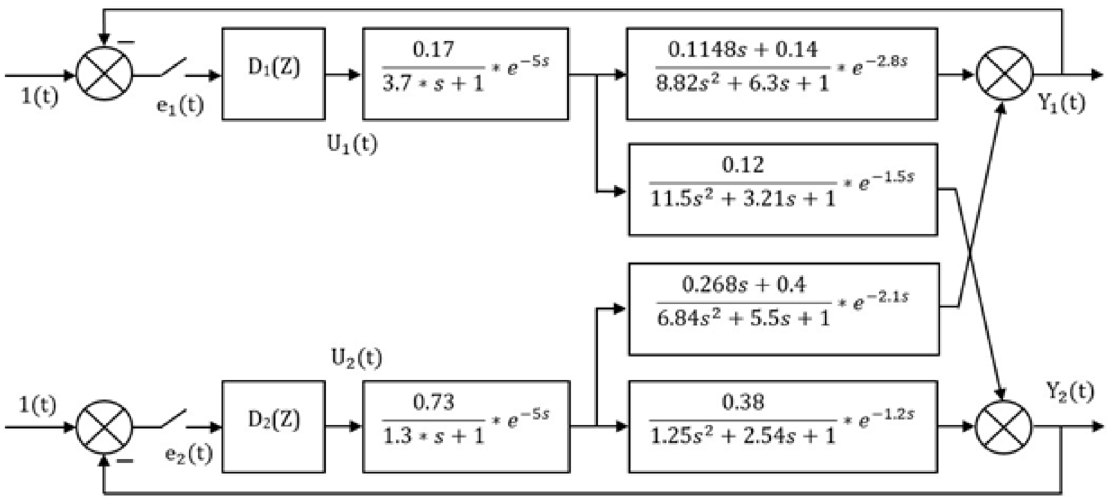


Figure. 2. Structural diagram of a digital control system.

Using the proposed approach, we determine the values of the control signal for each channel F by original variables U_i- after reducing similar terms, we obtain the following system of equations:

$$U_1 h_1 h_{k-1} + U_2 h_1 h_{k-2} + ... + U_{k-2} h_1 h_2 + U_{k-1} h_1^2 = Y^{зад} h_1 \qquad (17)$$

An analysis of this system shows that the squareness of criterion (4) ensures the existence of a unique solution for nonzero initial moments of the transition function.

The described control synthesis approach is convenient in that it is enough to have an object model in the form of a transition function, which is relatively easy to obtain as a result of active or pseudo-active experiments on the object, or as a result of statistical processing of data collected in a passive way. The developed algorithm is distinguished by the convenience of implementation in microprocessor tools.

3. Solution example. Consider the applicability of the algorithm for solving the problem of synthesis of the CSO block diagram, which is shown in fig.1.

It can be seen from the block diagram that the total order of the transfer function of the transmission channels is 12, which means that the process ends in 6 cycles.

Wherein $T = 5 sek$. a. Table 1.1

	1	2	3	4	5	6
U_1	11300.17	1275.96	482.68	36.88	-773.5	-773.5
Y_1	0	0.0273	0.5308	0.9181	1.0022	0.9999
U_2	0	4318.08	1398.82	-658.34	348.57	348.57
Y_2	0	0.7342	1.1962	1.1093	0.9956	1.0001

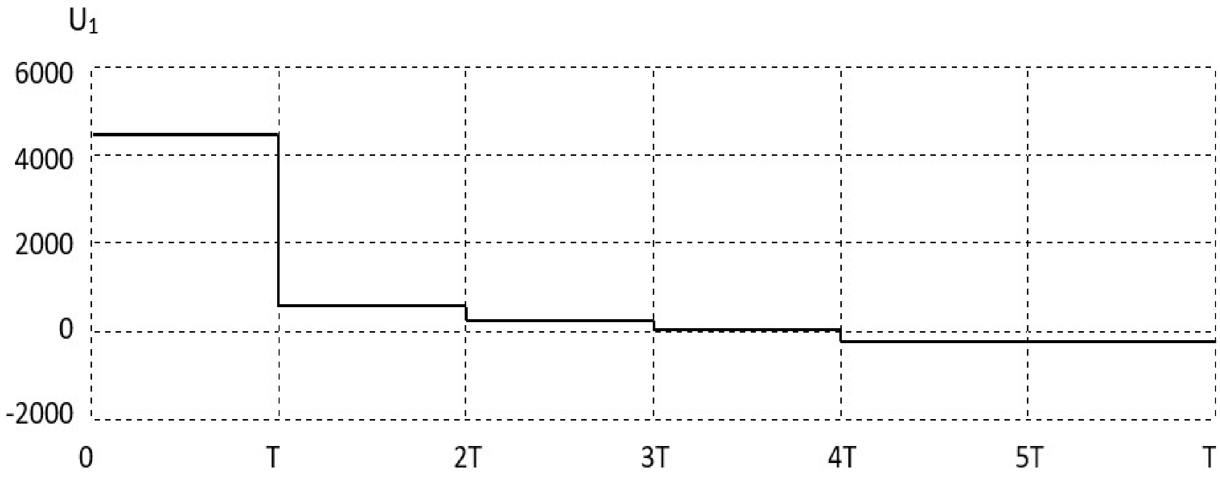


Figure. 3. Transient response of the drying process of the synthesized automatic control system.

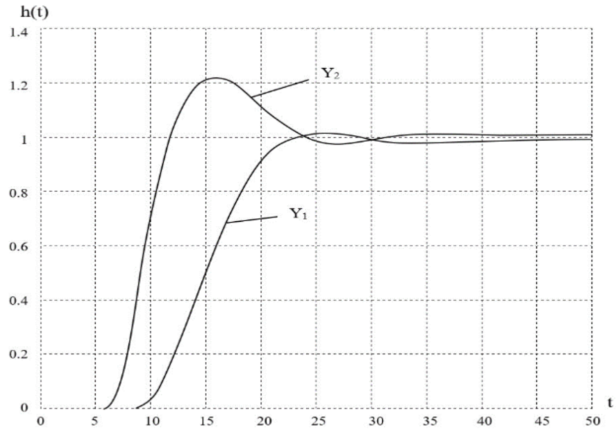


Figure. 4. Graph of the transient process of the automatic control system.

4. Conclusion

The results of a study on the creation of an algorithm for the synthesis of a discrete control system for a multidimensional dynamic object are given.. The possibility of using a hybrid synthesis method based on the joint use of a topological interpolation approach and methods of variational calculus is considered.The above studies have shown the possibility of using the developed synthesis algorithm for controlling dynamic objects, which allows minimizing the total squared deviation of the output values from the required ones. The resulting digital controller structures can be applied to control real technological processes, which improves the efficiency of the control system.

REFERENCES:

[1] Norman S. Nise. Control Systems Engineering. New York, John Wiley, 7 edition, 2015. – 944 p.

[2] Yusupbekov N.R., Muxamedov B.E., G‘ulomov Sh.M. Texnologik jarayonlarni boshqarish tizimlari. – Toshkent: «O‘qituvchi», 1997. -704 b.

[3] Unbehauen, H. Control Engineering. 3 Vols. Braunschweig, F. Vieweg & Sohn Verlagsgesellschaft. German. 2001. - 1273 p.

[4] Kim T., Maruta I., Sugie T. (2008) Robust PID controller tuning based on the constrained particle swarm optimization //Automatica. Vol. 44. № 4, P. 1104–1110.

[5] Pelusi D. (2012) PID and intelligent controllers for optimal timing performances of industrial actuators //International Journal of Simulation: Systems, Science and Technology, vol. 13, no. 2, pp. 65–71.

[6] Siddikov I.X., Sherbobyeva G.B., Rustamova M.B. (2020) Synthesis of terminal control system of discrete nonlinear objects with PWM modulation// Journal of Fhysics, Engineering and Technology. Vol. 5. Issue 1. January, P. 5089-5093.

[7] Malika Rustamova, Gulruh Sherboboyeva, Adkham Amonov, Khakberdi Jumanov, Shakhobiddin Faiziev and Tuygun Jumaboyev(2022) Development of models and algorithms for studying multi-dimensional systems with latitude-impulse modulation. AIP Conference Proceedings 2467, 020015 (2022); https://doi.org/10.1063/5.0092838 Published Online: 22 June 2022.

[8] Isamiddin Siddikov, Renata Izmaylovaand Malika Rustamova. Investigation of quantization cycle effect on control system stability. AIP Conference Proceedings 2432, 020006 (2022); https://doi.org/10.1063/5.0089475 Published Online: 16 June 2022