CONNECTIVITY OF MULTIVALUED REFLECTION GRAPHS IN COMPLEX DOMAINS

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ABSTRACT

In this article, a multi-valued reflection is created using derivative numbers in complex spaces. The dependency conditions of the reflection graph created using this collection are analyzed.

Keywords: derivative numbers, connectivity, monogeneity set.

INTRODUCTION

In the 70s of the 17th century, after I. Newton and G. Leibniz independently introduced the concept of derivative into science, differential and integral calculus were founded. By now, solving many practical problems leads to a wider and deeper study of exponential functions. Impairment analysis is one of the well-formed and rapidly developing branches of modern mathematics. It is clear from the term analysis that the object of its research is non-differentiable functions.The following decades were the years of rapid development of the theory, methods and applications of qualitative analysis. This was caused by three factors: the presence of theoretical and numerical methods that meet the needs of modern science and technology economy; the possibilities of modern computer technology. On the other hand, in the process of studying derivative numbers, religious numbers and their sets, multi-valued reflections appear. Multivalued reflections are often found in mathematics and related fields, theoretical physics, economics. Scientific results of Dini U. Menshov E.S., Trokhimchuk J.J. [3], Clark F, Demyaov V.F. can be noted in these areas of mathematics. in this field, associate professor f.m.f.n.Alikulov.E.O has conducted many researches in Uzbekistan. [5]

METHODS

Reflections formed using derivative numbers, methods of finding the connection of graphs, reflections with many complex variables, theory of multi-valued reflection using derivative numbers.

RESULT AND DISCUSSION

Let a continuous function $f(z)=u(x,y)+iv(x,y)$ be given in a domain D of the complex plane $\mathbb{C}_z$. For each point $z\in D$ consider the set

$$M_z(f)=\overline{\bigcap_{\varepsilon>0} M_\varepsilon(f;z)}$$

$$M_\varepsilon(f;z)=\left\{\frac{1}{z}\left(f(z+\Delta z)-f(z)\right), 0<|\Delta z|\le\varepsilon\right\}$$

Theorem1. [5] Let $f(z)=u(x,y)+iv(x,y)$ be a continuous and differentiable function in $D\subset\mathbb{C}_z$ then

$$W_\Phi=\{(z,\zeta): z\in D, \zeta\in M_z(f)\}$$

connected subset $\mathbb{C}^2(z,\zeta)$

Let $D\subset\mathbb{C}^n$ is continuous in the field $f:D\to\mathbb{C}$ is a given function. Every for the number $z=(z_1,z_2,...,z_n)\in D$ and $\varepsilon>0$ the following

$$M_1(f;z_1)=\left\{\frac{f(z_1+\Delta z_1,z_2,...,z_n)-f(z_1,z_2,...,z_n)}{\Delta z_1}, 0<|\Delta z_1|<\varepsilon\right\}$$

$$M_2(f;z_2)=\left\{\frac{f(z_1,z_2+\Delta z_2,...,z_n)-f(z_1,z_2,...,z_n)}{\Delta z_2}, 0<|\Delta z_2|<\varepsilon\right\}$$

$$M_n(f;z_n)=\left\{\frac{f(z_1,z_2,...,z_n+\Delta z_n)-f(z_1,z_2,...,z_n)}{\Delta z_n}, 0<|\Delta z_n|<\varepsilon\right\}$$

$$M(f;z_i)=\bigcup_{\varepsilon>0}\left\{\frac{f(z_1,z_2,...,z_i+\Delta z_i,...,z_n)-f(z_1,z_2,...,z_n)}{\Delta z_i}, 0<|\Delta z_i|<\varepsilon\right\}$$

Let's look at the collection.

Definition 1. A function f is a set of monogeneity at the point $z_i, i=1,2,..,n$, and is said to be the closure of the intersection of all sets $M(f;z_i)$ corresponding

$$M_{z_i}(f)=\overline{\bigcap_{\varepsilon>0} M_\varepsilon(f;z_i)}$$

to any $\varepsilon>0$, and is denoted as $M_{z_i}(f)$.So the function f is at the point $z\in D\subset\mathbb{C}^n$ that the set of monogeneity

$$\mathcal{M}_{z_i}(f)=\overline{\bigcap_{\varepsilon>0}\mathcal{M}_{\varepsilon}(f;z_i)}$$

This set is the function f at the the point z,in terms of variables overlaps with the singular derivative numbers.

Theorem. Let the function f be $\mathbb{C}$ $-$differentiable in the domain $D\subset\mathbb{C}^n$. Then $\Phi_f : z_i \to \mathcal{M}_{z_i}(f)$ is the graph of reflection

$G_\Phi=\{(z,h): z\in D, h\in\mathcal{M}_{z_i}(f)\}\subset\mathbb{C}^{n+1}_{(z,n)}$ -will be connected in space.

Proof. Let F be a open-closed subset of G_Φ and $\varnothing\neq F\neq G_\Phi$.Note that it consist of entire sets $\mathcal{M}_{z_i}(f)$.Let's put $A=\Phi_f^{-1}(F)$ .The connection of D implies $Fr(A)=\overline{A}\bigcap\overline{D\setminus A}\neq\varnothing$.Since the functions f_{z_i} and $f_{\overline{z}_i}$ in the first Baire class, the restrictions $f_{z_i}/Fr(A)$ $f_{\overline{z}_i}/Fr(A)$ everywhere have a set of points of continuity of the second category on $Fr(A)$.Therefore, there is a points z_0 of their simultaneous continuity.Since F and $G\setminus F$,and A and $D\setminus A$ satisfy the same assumptions,we can assume that $z_0\in A$ and therefore $z_0\in A\bigcap\overline{D\setminus A}$.

The set $A=\Phi_f^{-1}(F)$ contain the neighborhood of the point z_0 with recpect to many $Fr(A)$,since $\Phi\setminus Fr(A)$ is continuous at the point z_0 and F is neighborhood of $\Phi(z_0)=\mathcal{M}_{z_0}$.Therefore, there exist $\varepsilon>0$ such that from the conditions $|z_i-z_0|<\varepsilon, z_i\in Fr(A)$ it follows that $z_0\in A$. Let z_1 be a point of the set $D\setminus A$ such that $|z_i-z_0|<\varepsilon$ (In accordance with condition $z_0\in\overline{D\setminus A}$). Consequently $z_1\notin Fr(A)$,and therefore $z_1\in\overline{D\setminus A}$. Connect points z_0 and z_1 with the segment $[z_0,z_1]$.Let us choose a circle K centred at the point z_1 so that $\overline{K}\subset D\setminus\overline{A}$.Move K along the segment $[z_0,z_1]$ up to the first point of contact with the set $Fr(A)$.The point of contact will again be denoted by z_0.

If follows from the $\mathbb{C}$ $-$ differentiablity of the function that the set $\mathcal{M}_{z_0}(f)$ can be:1) a circle;2) a point.

Consider the case when $\mathcal{M}_{z_0}(f)$ is a circle. Let us prove that the distance $\mathcal{M}_{z_0}(f)$ and $\bigcup_{z\in K}\mathcal{M}_{z_i}(f)$ is equal to zero,and this will be the desired contradiction.

Let's assume that this distance is non-zero. Then if we project the sets $\bigcup_{z\in K}\mathcal{M}_{z_i}(f)$ and $\mathcal{M}_{z_0}(f)$ onto the plane $\mathbb{C}_\zeta$, then the distance between their projection will be positive.Let us enclose the circle $\mathcal{M}_{z_0}(f)$ in such a thin circular ring

T that it does intersect with $\bigcup_{z\in K} M_{z_i}(f)$. It is easy to see that for any location of $\bigcup_{z\in K} M_{z_i}(f)$ and $M_{z_0}(f)$,taking point $\zeta_0 \in T$ inside the circle $M_{z_0}(f)$ or outside it,you can ensurethat for displaying ψ carried out by the function $w = f(z) - \zeta_0 z = \psi(z)$. Inside K there necessarily existed points where the sign of the Jacobin $J_\psi(z)$ was opposite to the sign of the Jacobin J_ψ at the point z_0.Since in the case $J_\psi(z)$ inside K vanishes nowhere,then the mapping ψ is isolated and open at every point of K.i.e,it is an internal mapping of the circle K.By Stoilov's theorem [3] ,the values of $J_\psi(z)$ are constant everywhere in K.

It follows from the foregoing that we can assume $J_\psi(z), z \in K$ are positive.Then at the point z_0 the Jacobian $J_\psi(z)$ is negative.

The differential of the mapping ψ at the point z_0 is a linear mapping with a negative determinant. Therefore, it has two eigenvectors:one with positive eigenvalues,the other with negative ones.The line containing the first (second) vector will be denoted by $l^+(l^-)$. In this case, the specified linear mapping performs an oblique-mirror mapping with respect to the line l,in which all the lines parallel to l^- pass into themselves with opposite orientations.

Due to the fact that $J_\psi(z_0) \neq 0$,the boundary ∂K of the circle K passed to some curve of the w-plane under the mapping ψ,which has a certain tangent at the point $w_0 = \psi(z_0)$.Two cases are possible a) l coincides with l^+;b) l not the same as l^+.

In case a) we choose a pair of (sufficiently thin)vertical angles V with a common vertex w_0 and bisector l If the circle Δ centered at the point w_0 is sufficiently small, then only one of the sectors $\Delta \setminus V$ contain points of the image $\varphi(K)$.Let's denote it by Δ'.

In case b)we choose the vertical angles V so thin that do not intersect with l^+ (except for the point w_0) but otherwise the same as in case a).

We choose the circle k_0 at the point z_0 so that $\psi \setminus k_0$ is isolated, which can be done by virtue of the inequality $J_\psi(z_0) \neq 0$.Now we choose the circle Δ indicated above so that $\Delta \cap \psi(\partial k_0) \neq \varnothing$ Denote by z_0 the pre-image component $\psi^{-1}(\Delta)$ containing the point z_0 and $\delta' = \delta \cap K$.The region δ' may not be a normal region, i. e, its region may not coincide with the entire circle D.But it is easy to see that $\psi(\delta)$

contains the entire D. Indeed, the image of $\psi(\delta)$ contains points of the circle Δ' inside, and part of the boundary $\partial\delta'$ may not go only to the boundary $\partial\Delta'$,since in this case the $\overline{\delta}\cap\partial K$ (the common boundary δ' and K) is mapped into the angles V .this implies the assertion under consideration. In Δ' we consider a segment $[w_0, w']$ of radius r not lying on l^+.Let's prove that there is a simple δ' starting from the point z_0 and lying with ends δ' such that $\psi(\sigma)=[w_0, w_1]$ and ψ/σ is a homeomorphism.

The scheme of that proof of this fact repeats the proof of Stoilov's lemma $[11.p.139]$ about simple arc. For each $n=1,2,...$ we construct chains (Δ_n^ν) $\nu=0,1,2,....2^{\nu_n}$ closed circles Δ_n^ν covering the segment $[w_0, w']$.Namely a chain $(\Delta_1^\nu),\Delta=1,2,...,2^{\nu_n}$ we build as follows. We take an arbitrary circle $\Delta_1^0\subset\Delta$ centered at the point w_0 of radius $r<1$ such that the circle $\partial\Delta_1^0$ do intersects the segment $[w_0, w']$ at some point w''. Otre $[w'', w']$ cover with closed circles $\Delta_1^\nu,\nu=0,1,2,...,2^{\nu_1}$ so that:

1. $\Delta_1^\nu\cap V\neq\varnothing,\nu=1,2,...,2^{\nu_1}$
2. $\Delta_1^\nu\subset\Delta',\nu=1,2,...,2^{\nu_1}$

For $n>1$ chains (Δ_n^ν) are constructed in the same way, starting from arbitrary circles Δ_n^0 centers at w_0 and radiuses r_n less that $1/n$ covering respectively, the segments$\left[w^{(n+1)}, w'\right]$with conditions 1 and 2,as well as with the conditions:

3. $\Delta_n^p\cap\Delta_n^q=\varnothing$,if $|p-q|\geq 2, p,(q)=1,2,...,2^p,(2^q)$

4. $\Delta_{n+1}^\nu\subset\Delta_n^\nu$ if the segment enclosed in the house Δ_{n+1}^ν is part of the segment , $[w_0, w']$,concluded in Δ_n^ν for any ν and n.

5.for $n\to\infty$ the radius $r_n\to 0$.

To each chain (Δ_n^ν) we assign chains of regions (δ_n^ν) of such.so that for each $\nu>0$

δ_n^ν is a component of the pre-image $\psi^{-1}(\Delta_n^\nu)$.For this, in δ',first, we will put Δ_n^0 in correspondence with the δ_n^0containing the point z_0 such that $\psi(\delta_n^0)\subset\Delta_n^0$ to. It may also be an abnormal area, but it contains at least one point z_1 such that $\psi(z_1)=w_1$. Where w_1-is a point of the segment $[w_0, w']$.The set $\psi^{-1}(w_1)$ is closed and compact in δ_n^0.We surround each $z_1=\psi^{-1}(w_1)$ with a circle γ of such small radius that $\psi(\gamma)$ Δ_n^1

is inside and which is possible, since w_1 lies inside this region and the map ψ is continuous. Based on the Borel-Lebesgue lemma, one can choose a finite number of circles γ covering $\psi^{-1}(w_1)$.Then the number of regions $(\Delta_n^1 \cdot z_1)$ is finite. As δ_n^1 we take any of $(\Delta_n^1 \cdot z_1)$ and it will be the norm of δ_n^1 the floor, since it lies in δ'.Which each δ_n^1 we do the same as with δ_n^0,replacing the point z_1 with the point z_2. Thus, we obtain a finite number of chain (δ_n^ν) each of which has the properties indicated above. Next, following the scheme of the proof of Stoilov lemma, we obtain the sequence of chains $(\delta_n^1),(\delta_n^2),...,(\delta_n^\nu),....$ each of which is contained in all the previous ones and contain all subsequent ones. The set of such a sequence is a bounded continuum, and the intersection of σ these continuums is, as is well known, a continuum distinct from a point. Since chains (Δ_n^ν) tend to $[w_0, w']$ as $n \to \infty$,then $\psi(\sigma) \subset [w_0, w']$ and thus $\psi(\sigma) = [w_0, w']$.We now prove that ψ/σ is a homeomorphism. Let $z', z'' \in \sigma$ and $z' \neq z''$ be such that $\psi(z') = \psi(z'') = w$.By condition Δ_n^p,two regions Δ_n^p,a contain the point w:if $|p-q| \leq 2$ (for the same n) ,then the regions δ_n^ν containing z', z'' either coincide or are adjacent. Hence, they always form a continuum c_n,and the sets of the sequence $\{c_n\}$ are nested one into the other in exactly the same way as the sets of the sequence $\{\delta_n^\nu\}$.The intersection

$\bigcap_n c_n = c$ is a continuum, since it must contain the points z', z'' which by assumption are distinct. But $\psi(c)$ can only be a point of w,since $\psi(c_n)$ have no other common points except w.This contradicts the condition of the inner mapping that the continuums do not map to a single point, but since, on the other hand, the differensial of the mapping ψ is a linear mapping with a negative determinant, the image the part of the arc σ near z_0 is located on the other side l^+ and this is impossible due to the fact that ψ/σ is a homeomorphism. The obtained contradiction proves theorem in the case when $M_{z_0}(f)$ is a circle.

If $M_{z_0}(f)$ is a point far from $\bigcup_{z\in K} M_{z_i}(f)$,we introduce an auxiliary function $w = f(z) - \varepsilon \bar{z}$.For sufficiently small $\varepsilon > 0$,the circle $M_{z_0}(f)$ will also be far from $\bigcup_{z\in K} M_{z_i}(f)$,and it all comes back to the previous one. Using theorem, we prove a stronger assertion.

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