THE EQUATION OF A STRAIGHT LINE AND A PARABOLA FORMED AS A RESULT OF A SYMMETRIC TRANSLATION WITH RESPECT TO A STRAIGHT LINE $y = kx + b$.

Dilraboxon Mamatboyeva
Head teacher of Departmet of exact science,
The lyceum of Andijan State University,
Uzbekistan, Andijan

Feruza Mamajonova
Head teacher of Departmet of exact science,
The lyceum of Andijan State University,
Uzbekistan, Andijan

Dilnoza Qo‘shaqova
Tutor of Presidential school in Andijan, Uzbekistan.

Muslima Muhammadjonova
Student of Andijan State University,
Uzbekistan, Andijan

ABSTRACT

Everyone knows mathematics is the science that deals with the logic of shape, quantity and arrangement. Math is all around us, in everything we do. It is the building block for everything in our daily lives, including mobile devices, computers, software, architecture (ancient and modern), art, money, engineering and even sports.

In this paper, we study making symmetric shapes of any straight lines and parabolas. To do this, we use moving the origin of coordinate system and rotating it. And important thing is that we can find formulas for determining symmetric shapes of the given straight lines and giperbolas since we know coefficients of equations of the given straight lines and giperbolas.

Keywords: straight line, parabola, origin of coordinate, rotating, symmetric shapes.

MAIN RESULT

Now, it is time to introduce the content of the paper. We start with straight lines. We have two straight lines: $y = k_1x + b_1$ (1) and $y = k_2x + b_2$ (2). Where we have to transfer the straight line (1) symmetrically by the straight line (2).

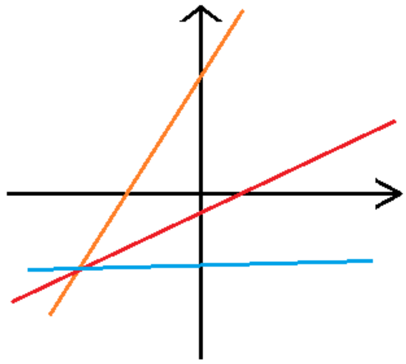

Figure 1

In Figure 1, the orange straight line is a given line (1) which has to be transferred symmetrically by the red one (2), and the blue one is formed. Our job is to find equation for the blue one. To do this, we rotate and move the origin of coordinate system as follows: transferring the origin of coordinate system to the point $(0; b_2)$ and rotating the coordinate system by α, where $tan\,\alpha = \frac{k_2-1}{k_2+1}$ (this will be shown how to obtain in the main part) which means we rotate the coordinate system until the slope of red straight line is $45°$ to the new coordinate system.

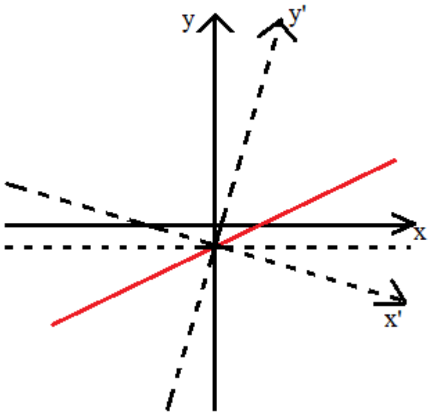

Figure 2

Here, the main idea is that if the graph of a function is transferred by $y = x$, it is equal to the graph of the inverse function of the given one (if it exists). Now, we move on main part.

First of all, we move the origin of coordinate system to the point $(0; b_2)$ by $Y = y - b_2$, $X = x$ (Figure 2). After that, equations (1) and (2) change to

$$\begin{cases} Y = k_1X + b_1 - b_2 \\ Y = k_2X \end{cases} \quad (3)$$

Next step is to change $Y = k_2X$ to $y' = x'$ by rotating coordinate system (Figure 2). For this, we use the following equations:

$$\begin{cases} Y = x' \sin\alpha + y' \cos\alpha \\ X = x' \cos\alpha - y' \sin\alpha \end{cases} \quad (4)$$

By putting (4) into the second equation of (3), we have

$$x' \sin\alpha + y' \cos\alpha = k_2(x' \cos\alpha - y' \sin\alpha)$$
$$y'(\cos\alpha + k_2 \sin\alpha) = x'(k_2 \cos\alpha - \sin\alpha)$$
$$y' = \frac{x'(k_2 \cos\alpha - \sin\alpha)}{(\cos\alpha + k_2 \sin\alpha)} \quad (5)$$

From (5), we can find α, because, we should make $y' = x'$. Thus,

$$\frac{k_2 \cos\alpha - \sin\alpha}{\cos\alpha + k_2 \sin\alpha} = 1$$
$$\tan\alpha = \frac{k_2 - 1}{k_2 + 1}$$

The next step is to put (4) into the first equation of (3):

$$x' \sin\alpha + y' \cos\alpha = k_1(x' \cos\alpha - y' \sin\alpha) + b_1 - b_2$$
$$y'(\cos\alpha + k_1 \sin\alpha) = x'(k_1 \cos\alpha - \sin\alpha) + b_1 - b_2$$

Therefore, we have the following equations:

$$\begin{cases} y'(\cos\alpha + k_1 \sin\alpha) = x'(k_1 \cos\alpha - \sin\alpha) + b_1 - b_2 \\ y' = x', \qquad \tan\alpha = \dfrac{k_2 - 1}{k_2 + 1} \end{cases}$$

And from here, we can find equation of the given straight line (1) transferring by the straight line (2) through finding inverse function found straight line:

$$x'(\cos\alpha + k_1 \sin\alpha) = y'(k_1 \cos\alpha - \sin\alpha) + b_1 - b_2$$
(6)

(6) is the equation we need to find. The last work is to go back to the first coordinate system by following equations:

$$\begin{cases} x' = Y \sin\alpha + X \cos\alpha \\ y' = Y \cos\alpha - X \sin\alpha \end{cases} \text{and} \begin{cases} X = x \\ Y = y - b_2 \end{cases}$$

$$(Y \cos\alpha - X \sin\alpha)(k_1 \cos\alpha - \sin\alpha) =$$
$$= (Y \sin\alpha + X \cos\alpha)(\cos\alpha + k_1 \sin\alpha) + b_2 - b_1$$
$$Y(k_1 \cos 2\alpha - \sin 2\alpha) = X(\cos 2\alpha + k_1 \sin 2\alpha) + b_2 - b_1$$

Now, we need $\cos 2\alpha$, $\sin 2\alpha$. We use $\tan\alpha = \frac{k_2 - 1}{k_2 + 1}$ to find $\cos 2\alpha$, $\sin 2\alpha$:

$$\sin 2\alpha = \frac{2 \tan\alpha}{1 + (\tan\alpha)^2} = \frac{k_2^2 - 1}{k_2^2 + 1}$$
$$\cos 2\alpha = \frac{1 - (\tan\alpha)^2}{1 + (\tan\alpha)^2} = \frac{2k_2}{k_2^2 + 1}$$

We obtain the following equation from above:

$$Y\left(k_1\cdot\frac{2k_2}{k_2^2+1}-\frac{k_2^2-1}{k_2^2+1}\right)=X\left(\frac{2k_2}{k_2^2+1}+k_1\cdot\frac{k_2^2-1}{k_2^2+1}\right)+b_2-b_1$$

$$Y(2k_1k_2-k_2^2+1)=X(2k_2+k_1k_2^2-k_1)+(b_2-b_1)(k_2^2+1)$$

And finally,

$$(y-b_2)(2k_1k_2-k_2^2+1)=x(2k_2+k_1k_2^2-k_1)+(b_2-b_1)(k_2^2+1)$$

$$y=\frac{x(2k_2+k_1k_2^2-k_1)+(b_2-b_1)(k_2^2+1)}{(2k_1k_2-k_2^2+1)}+b_2 \qquad (7)$$

(7) is the equation of the straight line we are looking for.

We move on the second part of the paper. Likewise, in this part, we transfer parabolas by any straight lines using the method like above.

$$\begin{cases} y=ax^2+bx+c \\ y=kx+l \end{cases} \qquad (8)$$

We have a parabola with coefficients a, b, c and a straight line with k, l. First, we transfer the origin of coordinate system: $Y=y-l$ and $X=x$. And we obtain

$$\begin{cases} Y=aX^2+bX+c-l \\ Y=kX \end{cases}$$

Now, we use transformation (4) and equation $tan\,\alpha=\frac{k-1}{k+1}$:

$$\begin{cases} x'\,sin\,\alpha+y'\,cos\,\alpha=a(x'^2\,cos^2\,\alpha-x'y'\,sin\,2\alpha+y'^2\,sin^2\,\alpha)+ \\ +b(x'\,cos\,\alpha-y'\,sin\,\alpha)+c-l \\ y'=x', \qquad tan\,\alpha=\dfrac{k-1}{k+1} \end{cases}$$

Again, we return to the actual coordinate system (x, y) by

$$\begin{cases} x'=Y\,sin\,\alpha+X\,cos\,\alpha \\ y'=Y\,cos\,\alpha-X\,sin\,\alpha \end{cases} \text{and} \begin{cases} X=x \\ Y=y-l \end{cases}$$

formulas. After some calculations, we obtain the equation what we are looking for:

$$4a(y-l)^2k^4-4akx(y-l)(k^2-1)+ax^2(k^2-1)^2+$$
$$+(y-l)(2bk(k^2+1)-k^4+1)-x\big(b(k^4-1)+2k(k^2+1)\big)+$$
(9)
$$+(c-l)(k^2+1)^2=0$$

(9) equation is our main goal. Finally, we have known how to transfer a straight line and a parabola by any straight line and have obtained equations (7) and (9).

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