BA’ZI BIR NILPOTENT ZINBIEL ALGEBRALARIDA ROTA – BAXTER OPERATORLARI

M.A. Qodirova
Andijon davlat universiteti

E.U. Tojiboyev
Andijon davlat universiteti

ANNOTATSIYA

Ushbu maqolada noassotsiativ algebra boʻlgan 4 oʻlchamli nilpotent Zinbiel algebralarida vazni λ boʻlgan Rota-Baxter operatorlari oʻrganilgan. Bundan tashqari ba’zi 4 oʻlchamli nilpotent Zinbiel algebrlarida vazni nol boʻlgan Rota-Baxter operatorlari matritsalari qurilgan. Bunda 4 oʻlchamli nilpotent Zinbiel algebralarining 2010-yilda chop etilgan “Zinbiel algebralarining ayrim sinflari tasnifi” nomli maqoladagi tasnifidan foydalanilgan.

Kalit soʻzlar: noassotsiativ algebra, vazn, operator, maydon, akslantirish.

ROTA-BAXTER OPERATORS IN SOME NILPOTENT ZINBIEL ALGEBRAS

ABSTRACT

In this article studied Rota-Baxter operators of weight λ in 4-dimensional nilpotent non-associative Zinbiel algebras. In addition, for some 4-dimensional nilpotent Zinbiel algebras, matrices of Roth-Bachter operators with zero weight are constructed. Here we used the classification of 4-dimensional nilpotent Zinbiel algebras from the article “Classification of some classes of Zinbiel algebras” published in 2010.

Keywords: non-associative algebra, weight, operator, field, mapping.

Rota – Baxter algebrasi ehtimollar nazariyasidagi ba’zi bir masalalarni yechishdan kelib chiqqan boʻlib, matematika va fizikaning koʻplab sohalarida oʻz tadbigʻiga ega, jumladan sonlar nazariyasi, kvazisimmetrik funksiyalar, Li algebralari va Yang – Baxter tenglamalari. Rota – Baxter operatorlari Baxter tomonidan ehtimollar nazariyasidagi [3] analitik formulani yechish uchun kiritilgan. Bu operatorlar, matematika va matematik fizikaning boshqa sohalariga ham aloqador [4]. [1] ishda 3-

o‘lchamli nilpotent assotsiativ algebralar uchun Rota-Bakster, Reynold va Nijenhuis operatorlari tasnif qilingan.

F maydon ustida A assotsiativ algebraning Rota – Baxter operatori deb quyidagi tenglikni qanoatlantiruvchi $P: A \to A$ chiziqli akslantirishga aytiladi:

$$P(x)P(y) = P(xP(y) + P(x)y + \lambda xy), \quad \forall x, y \in A, \quad \lambda \in F$$

Takidlab o‘tish kerakki, agar P – $\lambda \neq 0$ vaznga ega Rota – Baxter operatori bo‘lsa, u holda λ^{-1} P – vazni 1 ga teng Rota – Baxter operatori.

Ushbu ishda biz ba’zi bir 4 – o‘lchamli nilpotent Zinbiel algebralarida Rota – Baxter operatorlarini tasnifladik.

P operatorning matritsasi – $\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$

λ=0 bo‘lgan xol

$$P(x)P(y) = P(xP(y) + P(x)y)$$

Quyida ba’zi bir 4 – o‘lchamli nilpotent Zinbiel algebralar [2] ishda keltirilgan:

α_1: $e_1 \circ e_1 = e_3$, $e_1 \circ e_2 = e_4$, $e_1 \circ e_3 = e_4$, $e_3 \circ e_1 = 2e_4$.

α_2: $e_1 \circ e_1 = e_3$, $e_1 \circ e_3 = e_4$, $e_2 \circ e_2 = e_4$, $e_3 \circ e_1 = 2e_4$.

α_3: $e_1 \circ e_1 = e_2, e_1 \circ e_2 = e_3, e_2 \circ e_1 = 2e_3$, $e_1 \circ e_3 = e_4$, $e_2 \circ e_2 = 3e_4$,

$e_3 \circ e_1 = 3e_4$;

α_4: $e_1 \circ e_2 = e_3$, $e_1 \circ e_3 = e_4$, $e_2 \circ e_1 = -e_3$, $e_2 \circ e_2 = e_4$.

Teorema 1. 4 – o‘lchamli α_1 nilpotent Zinbiel algebrasida Rota – Baxter operatori matritsalari quyidagicha bo‘ladi:

Algebra	Rota Baxter operatorlari λ=0 bo‘lgan xol	Cheklovlar
α_1	$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & 0 \end{pmatrix}$	$a_{11} = a_{44}$
	$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ -\frac{1}{3}a_{21} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$	$a_{11} \neq a_{44}$; $a_{11} = 0$

	$P_3 = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{31} & 0 & \frac{1}{2}a_{11} & 0 \\ a_{41} & a_{42} & a_{31} & \frac{1}{3}a_{11} \end{pmatrix}$	$a_{11} \neq a_{44};\ a_{11} \neq 0;$ $a_{43} = a_{31}$
	$P_4 = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ 3(a_{43} - a_{31}) & 0 & 0 & 0 \\ a_{31} & 0 & \frac{1}{2}a_{11} & 0 \\ a_{41} & a_{42} & a_{43} & \frac{1}{3}a_{11} \end{pmatrix}$	$a_{11} \neq a_{44};\ a_{11} \neq 0;$ $a_{43} \neq a_{31}.$

Teorema 2. 4 – o‘lchamli α_2 nilpotent Zinbiel algebrasida Rota – Baxter operatori matritsalari quyidagicha bo‘ladi:

Algebra	Rota Baxter operatorlari λ=0 bo‘lgan xol	Cheklovlar
α_2	$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & 0 \end{pmatrix}$	$a_{22} = a_{44}$
	$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$	$a_{22} \neq a_{44};\ a_{11} = 0;$ $a_{22} = 0.$
	$P_3 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & \frac{1}{2}a_{22} \end{pmatrix}$	$a_{22} \neq a_{44};\ a_{11} = 0;$ $a_{22} \neq 0.$
	$P_4 \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{31} & 0 & \frac{1}{2}a_{11} & 0 \\ a_{41} & a_{42} & a_{31} & \frac{1}{3}a_{11} \end{pmatrix}$	$a_{22} \neq a_{44};\ a_{11} \neq 0;$ $a_{22} = 0.$
	$P_5 \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ 0 & \frac{2}{3}a_{11} & 0 & 0 \\ a_{31} & \frac{1}{4}a_{11} & \frac{1}{2}a_{11} & 0 \\ a_{41} & a_{42} & a_{31} & \frac{1}{3}a_{11} \end{pmatrix}$	$a_{22} \neq a_{44};\ a_{11} \neq 0;$ $a_{22} \neq 0.$

Teorema 3. 4 – o'lchamli α_3 nilpotent Zinbiel algebrasida Rota – Baxter operatori matritsalari quyidagicha bo'ladi:

Algebra	Rota Baxter operatorlari λ=0 bo'lgan xol	Cheklovlar
α_3	P_1: $a_{11} = a_{12} = a_{13} = a_{14} = a_{21} = a_{22} = a_{23} = a_{24} = 0,$ $a_{34} = a_{44} = 0$ $a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} = 0$; $a_{21} = 0$; $a_{44} = 0.$
	P_2: $a_{11} = a_{12} = a_{13} = a_{14} = a_{21} = a_{22} = a_{23} = a_{24} = 0;$ $a_{31} = a_{32} = a_{33} = a_{34} = 0$ $a_{41}, a_{42}, a_{43}, a_{43}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} = 0$; $a_{21} = 0$; $a_{44} \neq 0.$
	P_3: $a_{11} = a_{12} = a_{13} = a_{14} = a_{22} = a_{23} = a_{24} = a_{33} = 0,$ $a_{34} = a_{44} = 0,$ $a_{43} = a_{21},$ $a_{21}, a_{31}, a_{32}, a_{41}, a_{42}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} = 0$; $a_{21} \neq 0$.
	P_4: $a_{11} = a_{12} = a_{13} = a_{14} = a_{21} = a_{23} = a_{24} = 0$, $a_{31} = a_{32} = a_{33} = a_{34} = a_{43} = 0;$ $a_{44} = \frac{1}{2}a_{22};$ a_{22}, a_{41}, a_{42} – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} = 0$; $a_{21} = 0.$
	P_5: $a_{11} = a_{12} = a_{13} = a_{14} = a_{23} = a_{24} = a_{33} = a_{34} = 0;$ $a_{31} = -\frac{3a_{21}^2}{a_{22}}$, $a_{32} = -3a_{21}$, $a_{43} = 3a_{21}$, $a_{44} = \frac{1}{2}a_{22};$ $a_{21}, a_{22}, a_{41}, a_{42}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} = 0$; $a_{21} \neq 0.$
	P_6: $a_{12} = a_{13} = a_{14} = a_{21} = a_{23} = a_{24} = a_{31} = a_{32} = 0,$ $a_{34} = a_{42} = a_{43} = 0;$ $a_{11} = 3a_{33}$, $a_{44} = \frac{1}{2}a_{22};$ a_{22}, a_{33}, a_{41} – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} \neq 0$; $a_{21} = 0$; $a_{31} = 0.$
	P_7: $a_{12} = a_{13} = a_{14} = a_{21} = a_{23} = a_{24} = a_{32} = a_{34} = a_{43} = 0;$ $a_{11} = 3a_{33}$, $a_{42} = \frac{3}{2}a_{31}$, $a_{44} = \frac{1}{2}a_{22};$ $a_{22}, a_{31}, a_{33}, a_{41}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} \neq 0$; $a_{21} = 0$; $a_{31} \neq 0.$
	P_8: $a_{12} = a_{13} = a_{14} = a_{23} = a_{24} = a_{31} = a_{34} = a_{42} = 0$; $a_{11} = 3a_{33}$, $a_{32} = a_{43} = a_{21}$, $a_{44} = \frac{1}{2}a_{22};$ $a_{21}, a_{22}, a_{33}, a_{41}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} \neq 0$; $a_{21} \neq 0$; $a_{31} = 0.$
	P_9: $a_{12} = a_{13} = a_{14} = a_{23} = a_{24} = a_{34} = 0$; $a_{11} = 3a_{33}$, $a_{32} = a_{43} = a_{21}$, $a_{42} = \frac{3}{2}a_{31}$, $a_{44} = \frac{1}{2}a_{22};$ $a_{21}, a_{22}, a_{31}, a_{33}, a_{41}$ – lar, cheklovlarga bog'liq holda ihtiyoriy.	$a_{22} \neq 0$; $a_{33} \neq 0$; $a_{21} \neq 0$; $a_{31} \neq 0.$

Teorema 4. 4 – o'lchamli α_4 nilpotent Zinbiel algebrasida Rota – Baxter operatori matritsalari quyidagicha bo'ladi:

Algebra	Rota Baxter operatorlari λ=0 bo‘lgan xol	Cheklovlar
	P_1: $a_{11}=a_{12}=a_{13}=a_{14}=a_{21}=a_{22}=a_{23}=a_{24}=0$, $a_{34}=a_{44}=0$ $a_{31},a_{32},a_{33},a_{41},a_{42},a_{43}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}=a_{44}$; $a_{12}=0$.
	P_2: $a_{11}=a_{13}=a_{14}=a_{21}=a_{22}=a_{23}=a_{24}=0$, $a_{31}=a_{32}=a_{33}=a_{34}=a_{44}=0$ $a_{12},a_{41},a_{42},a_{43}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}=a_{44}$; $a_{12}\neq 0$; $a_{22}=0$.
	P_3: $a_{11}=a_{13}=a_{14}=a_{21}=a_{23}=a_{24}=a_{31}=a_{33}=a_{34}=0$, $a_{43}=a_{44}=0$; $a_{32}=-\frac{a_{22}^2}{a_{12}}$; $a_{12},a_{22},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}=a_{44}$; $a_{12}\neq 0$; $a_{22}\neq 0$.
	P_4: $a_{11}=a_{12}=a_{13}=a_{14}=a_{21}=a_{22}=a_{23}=a_{24}=0$, $a_{31}=a_{32}=a_{33}=a_{34}=0$; $a_{41},a_{42},a_{43},a_{44}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}=0$; $a_{22}=0$; $a_{11}=0$.
	P_5: $a_{13}=a_{14}=a_{21}=a_{22}=a_{23}=a_{24}=a_{31}=a_{32}=0$, $a_{33}=a_{34}=a_{43}=a_{44}=0$; $a_{11},a_{12},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}=0$; $a_{22}=0$; $a_{11}\neq 0$.
	P_6: $a_{11}=a_{12}=a_{13}=a_{14}=a_{21}=a_{23}=a_{24}=a_{31}=a_{32}=0$, $a_{33}=a_{34}=a_{43}=0$; $a_{44}=\frac{1}{2}a_{22}$; a_{22},a_{41},a_{42} − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}=0$; $a_{22}\neq 0$; $a_{33}=0$.
α_4	P_7: $a_{12}=a_{13}=a_{14}=a_{21}=a_{23}=a_{24}=a_{31}=a_{32}=0$, $a_{34}=a_{43}=0$; $a_{22}=\frac{1}{2}a_{11}$, $a_{33}=\frac{1}{3}a_{11}$, $a_{44}=\frac{1}{4}a_{11}$; a_{11},a_{41},a_{42} − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}=0$; $a_{22}\neq 0$; $a_{33}\neq 0$.
	P_8: $a_{12}=a_{13}=a_{14}=a_{22}=a_{23}=a_{24}=a_{32}=a_{33}=0$, $a_{34}=a_{43}=a_{44}=0$; $a_{31}=\frac{a_{21}^2}{a_{11}}$; $a_{11},a_{21},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}\neq 0$; $a_{33}=a_{44}$; $a_{12}=0$.
	P_9: $a_{13}=a_{14}=a_{23}=a_{24}=a_{33}=a_{34}=a_{43}=a_{44}=0$; $a_{22}=\frac{a_{12}a_{21}}{a_{11}}$, $a_{31}=-\frac{a_{21}^2}{a_{11}}$, $a_{32}=-\frac{a_{12}a_{21}^2}{a_{11}^2}$; $a_{11},a_{12},a_{21},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}\neq 0$; $a_{33}=a_{44}$; $a_{12}\neq 0$; $a_{43}=0$.
	P_{10}: $a_{13}=a_{14}=a_{23}=a_{24}=a_{33}=a_{34}=a_{44}=0$; $a_{21}=-\frac{a_{11}^2}{a_{12}}$, $a_{22}=-a_{11}$, $a_{31}=-\frac{3a_{11}^3}{a_{12}^2}$, $a_{32}=-\frac{a_{11}^2}{a_{12}}$; $a_{11},a_{12},a_{41},a_{42},a_{43}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}\neq 0$; $a_{33}=a_{44}$; $a_{12}\neq 0$; $a_{43}\neq 0$.
	P_{11}: $a_{11}=a_{12}=a_{13}=a_{14}=a_{23}=a_{24}=a_{33}=a_{34}=0$; $a_{31}=-\frac{2a_{21}^2}{a_{22}}$, $a_{32}=-2a_{21}$, $a_{43}=-\frac{1}{2}a_{21}$, $a_{44}=\frac{1}{2}a_{22}$; $a_{21},a_{22},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}\neq 0$; $a_{33}\neq a_{44}$; $a_{33}=0$.
	P_{12}: $a_{12}=a_{13}=a_{14}=a_{23}=a_{24}=a_{34}=0$; $a_{22}=\frac{1}{2}a_{11}$, $a_{31}=\frac{4a_{21}^2}{3a_{11}}$, $a_{32}=-2a_{21}$, $a_{43}=-\frac{1}{6}a_{21}$, $a_{44}=\frac{1}{4}a_{11}$; $a_{21},a_{22},a_{41},a_{42}$ − lar, cheklovlarga bog‘liq holda ihtiyoriy.	$a_{11}\neq a_{44}$; $a_{21}\neq 0$; $a_{33}\neq a_{44}$; $a_{33}\neq 0$.

FOYDALANILGAN ADABIYOTLAR RO‘YXATI: (REFERENCES)

1. N.G.Abdujabborov, I.A.Karimjanov and M.A.Kodirova. Rota – type operators on 3 – dimensional nilpotent associative algebras. Communications in Mathematics 29 (2021) 227–241

2. J.Q.Adashev, B.A.Omirov and A.Kh.Khudoyberdiyev. Classifications of some classes of Zinbiel algebras. Journal of Generalized Lie Theory and Applications Vol. 4 (2010), Article ID S090601, 10 pages

3. G. Baxter: An analytic problem whose solution follows from a simple algebraic identity. Pac. J. Math. 10 (1960) 731–742.

4. X. Tang, Y. Zhang, and Q. Sun.: Rota-Baxter operators on 4-dimensional complex simple associative algebras. Appl. Math. Comput. 229 (2014) 173–186.