TRIGONOMETRY AND MODEL SIMPLE HARMONIC MOTION

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ABSTRACT

A mere snippet of a song from the past can trigger vivid memories, including emotions ranging from unabashed joy to deep sorrow. Trigonometric functions describe the pitch, loudness, and quality of a music notes. In this article we will learn how trigonometry models harmonic motion.

Keywords: trigonometry, amusia, sine, cosine, frequency, amplitude, Harmonic motion, waves, equilibrium**.**

Simple Harmonic Motion: Because of their periodic nature, trigonometric functions are used to model phenomena that occur again and again. This includes vibratory or oscillatory motion, such as the motion of a vibrating guitar string, the swinging of a pendulum, or the bobbing of an object attached to a spring. Trigonometric functions are also used to describe radio waves from your favorite FM station, television waves from your not- to-be-missed weekly sitcom, and sound waves from your most-prized CDs.

To see how trigonometric functions are used to model vibratory motion, consider this: A ball is attached to a spring hung from the ceiling. You pull the ball down 4 inches and then release it. If we neglect the effects of friction and air resistance, the ball will continue bobbing up and down on the end of the spring. These up-and-down oscillations are called simple harmonic motion.

To better understand this motion, we use a d-axis, where d represents distance. This axis is shown in Figure 1. On this axis, the position of the ball before you pull it down is d = 0. This rest position is called the equilibrium position. Now you pull the ball down 4 inches to d = -4 and release it. Figure 2 shows a sequence of "photographs" taken at one-second time intervals illustrating the distance of the ball from its rest position, d.

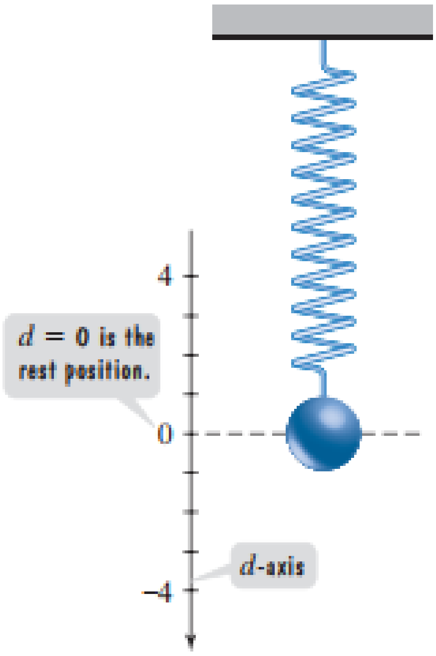


Figure 1. Using a d-axis to describe a ball's distance from its rest position

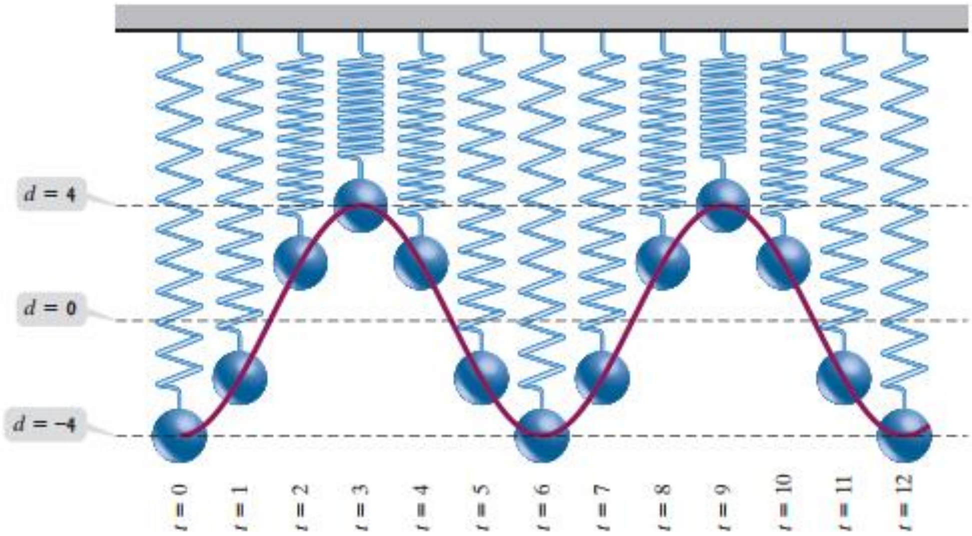


Figure 2. A sequence of "photographs" showing the bobbing ball's distance from the rest position taken at one-second intervals

The curve in **Figure 2** shows how the ball's distance from its rest position changes over time. The curve is sinusoidal and the motion can be described using a cosine or a sine function.

Simple Harmonic Motion: An object that moves on a coordinate axis is in simple harmonic motion if its distance from the origin, d, at time t is given by either

d=a $\cos \omega t$ **or** **d=a** $\sin \omega t$**.**

The motion has amplitude |a| , the maximum displacement of the object from its rest position. The period of the motion is $\frac{2\pi}{\omega}$,where ω>0. The period gives the time it takes for the motion to go through one complete cycle.

In describing simple harmonic motion, the equation with the cosine function, $d=a\cos\omega t$, is used if the object is at its greatest distance from rest position, the origin, at t = 0. By contrast, the equation with the sine function, $d=a\sin\omega t$., is used if the object is at its rest position, the origin, at t = 0.

Exp1. Finding an Equation for an Object in simple Harmonic Motion: A ball on a spring is pulled 4 inches below its rest position and then released. The period of the motion is 6 seconds. Write the equation for the ball's simple harmonic motion.

Solution: We need to write an equation that describes d, the distance of the ball from its rest position, after t seconds. (The motion is illustrated by the "photo" sequence in Figure 2.) When the object is released (t = 0), the ball's distance from its rest position is 4 inches down. Because it is down 4 inches, d is negative: When t = 0, d = -4. Notice that the greatest distance from rest position occurs at t = 0. Thus, we will use the equation with the cosine function,

$$d=a\cos\omega t$$

to model the ball's simple harmonic motion. Now we determine values for a and ω. Recall that |a| is the maximum displacement. Because the ball is initially below rest position, a = -4. The value of ω in $d=a\cos\omega t$ can be found using the formula for the period.

$\text{period} = \frac{2\pi}{\omega}=6$ we are given that the period of the motion is 6 seconds.

$2\pi = 6\omega$ Multiply both sides by ω.

$\omega=\frac{2\pi}{6}=\frac{\pi}{3}$ Divide both sides by 6 and solve for ω.

We see that $a = -4$ and $\omega =\frac{\pi}{3}$. Substitute these values into $d=a\cos\omega t$. The equation for the ball's simple harmonic motion is

$$d=-4\cos\frac{\pi}{3}t.$$

Check Point: A ball on a spring is pulled 6 inches below its rest position and then released. The period for the motion is 4 seconds. Write the equation for the ball's simple harmonic motion.

The period of the harmonic motion in Example 1 was 6 seconds. It takes 6 seconds for the moving object to complete one cycle. Thus, $\frac{1}{6}$ of a cycle is completed every second. We call $\frac{1}{6}$ the frequency of the moving object. Frequency describes the number of complete cycles per unit time and is the reciprocal of the period.

Frequency of an Object in Simple Harmonic Motion: An object in simple harmonic motion given by

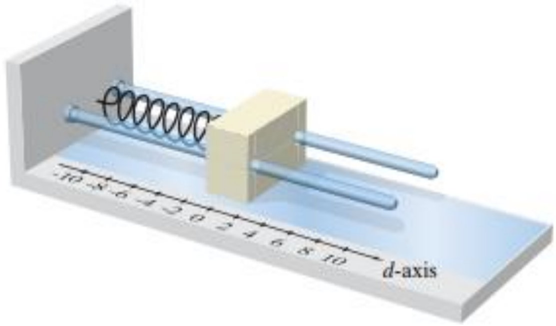
$$d=a\cos\omega t \quad \text{or} \quad d=a\sin\omega t.$$

has frequency f given by $f=\frac{\omega}{2\pi}$, $\omega>0$.

Equivalently, $f=\frac{1}{\text{period}}$

Exp2. Analyzing Simple Harmonic Motion: Figure 3. shows a mass on a smooth surface attached to a spring. The mass moves in simple harmonic motion described by $d=10\cos\frac{\pi}{6}t$,

FIGURE 3 A mass attached to a spring moving in simple harmonic motion



where t is measured in seconds and d in centimeters. Find:

a. the maximum displacement

b. the frequency

c. the time required for one cycle

Solution: We begin by identifying values for a and ω.

$$d=10\cos\frac{\pi}{6}t$$

The form of this equation is $d=a\cos\omega t$

with a=10 and $\omega=\frac{\pi}{6}$.

a. The maximum displacement from the rest position is the amplitude. Because a = 10, the maximum displacement is 10 centimeters.

b. The frequency, f, is

$$f=\frac{\omega}{2\pi}=\frac{\frac{\pi}{6}}{2\pi}=\frac{\pi}{6}\cdot\frac{1}{2\pi}=\frac{1}{12}.$$

The frequency is $\frac{1}{12}$ cycle (or oscillation) per second.

c. The time required for one cycle is the period.

$$\text{period}=\frac{2\pi}{\omega}=\frac{2\pi}{\frac{\pi}{6}}=12$$

The time required for one cycle is 12 seconds. This value can also be obtained by taking the reciprocal of the frequency in part (b).

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