TEKISLIKDA MOMENTLI ELASTIKLIK NAZARIYASI SISTEMASI ECHIMI UCHUN SOMILIAN - BETTI FORMULASI

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ANNOTASIYA

Bu ishda momentli elastikliklik nazariyasi tenglamalari sistemasi yechimini fazoda yechim va uning kuchlanishi soha chegarasining musbat o‘lchovli qismida berilganda sohaning ichiga topish masalasi qaraladi. Bunday masalaga Koshi masalasi deyiladi. Qaralayotgan masalaning yechimi mavjudligi kriteriyasi keltiriladi.

Kalit so‘zlar: Momentli elastikliklik nazariyasi, Karleman funksiyasi, Karleman matritsasi, Somilion-betti, Koshi masalasi.

KIRISH

Bu ish elastiklik nazariyasi tenglamalari sistemasi yechimini tekislikdagi chegaralanmagan sohada chegaraning qismida uning berilgan qiymatlari va uning kuchlanishi qiymatlari bo‘yicha davom ettirish masalasi, ya’ni elastiklik nazariyasi tenglamalari sistemasi uchun Koshi masalasi o‘rganiladi.

Bu ishda asosan chegaralanmagan sohada mos ravishda chegaralanmagan yechim holida Karleman matritsasini qurish yo‘llari o‘rganilgan. Karleman matritsasini qurishda bundan oldin qaralgan holdan farqli ravishda maxsuslik tartibi katta bo‘lgan yadro holida Karleman matritsasini mustaqil ravishda qurildi va shu holda regulyarlashgan yechim bilan aniq yechim orasidagi farq baholangan.

Usullari.

$x=(x_1,x_2)$va$y=(y_1,y_2)$ nuqtalar E^2- ikki o‘lchovli evklid fazosidan olingan bo‘lsin va D elastik muhit E^2 da bo‘lakli - silliq ∂D chiziq bilan chegaralangan sohadan iborat bo‘lsin, $S-\partial D$ ning silliq qismi.

Dsohada bir jinsli momentli elastiklik nazariyasi tenglamalari sistemasi

$$\begin{cases}(\mu+\alpha)\Delta u+(\lambda+\mu-\alpha)graddivu+2\alpha\, rotw+\rho\theta^2u=0,\\(\nu+\beta)\Delta w+(\varepsilon+\nu-\beta)graddivw+2\alpha\, rotu-4\alpha w+j\theta^2w=0,\end{cases}\tag{1}$$

berilgan bo‘lsin. Bu yerda $U(x)=(u_1(x),u_2(x),w_1(x),w_2(x))=(u(x),w(x))$ sistemaning yechimi, Δ- Laplas operatori, $\lambda,\mu,\nu,\beta,\varepsilon,\alpha$ elastik muhitni xarakterlaydigan sonlar bo‘lib, quyidagi shartlarni qanoatlantiradi $\mu>0, 3\lambda+2\mu>0,\ \alpha>0,\ \varepsilon>0,\ 3\varepsilon+2\nu>0,\ \beta>0,\ j>0,\ \rho>0,\ \theta\in R^1$.

1-ta'rif. D da aniqlangan φ funksiyani D da regulyar deb ataymiz, agar $\varphi\in C^1(\overline{D})\cap C^2(D)$ va φ- funksiyaning dekart koordinatalari bo‘yicha barcha ikkinchi tartibli hosilalari D sohada integrallanuvchi bo‘lsa.

Bundan keyin ∂D ni chekli D sohani chegaralovchi yopiq silliq egri chiziq deb hisoblaymiz.

1-teorema. D sohada (1) tenglamaning har qanday regulyar yechimi ushbu

$$U(x)=\int_{\partial D}(\Psi(y,x)\{T(\partial_y,n)U(y)\}-\{T(\partial_y,n)\Psi(y,x)\}^{*}U(y))ds_y,\ x\in D, \quad (2)$$

formula bilan beriladi. Bu yerda «*» belgi transponirlangan matritsani bildiradi.

Formulada keltirilgan $\Psi(y,x)$- momentli statik elastiklik nazariyasi tenglamalari sistemasining fundamental yechimlar matritsasi bo‘lib, u quyidagicha beriladi [1]:

$$\Psi(y,x)=\left\|\begin{matrix}\Psi^{(1)}(y,x) & \Psi^{(2)}(y,x)\\ \Psi^{(3)}(y,x) & \Psi^{(4)}(y,x)\end{matrix}\right\|,$$

bunda $\Psi^{(i)}(y,x)=\left\|\Psi^{(i)}_{kj}(y,x)\right\|_{2\times 2},\quad i=1,2,3,4,$

$$\Psi^{(1)}_{kj}(y,x)=\sum_{l=1}^{4}(\delta_{kj}\alpha_l+\beta_l\frac{\partial^2}{\partial x_k\partial x_j})\frac{e^{i\lambda_m|y-x|}}{|y-x|},\quad k,j=1,2,$$

$$\Psi^{(2)}_{kj}(y,x)=\Psi^{(3)}_{kj}(y,x)=\frac{2\alpha}{\mu+\alpha}\sum_{l=1}^{4}\sum_{p=1}^{2}\varepsilon_l\varepsilon_{kjp}\frac{\partial}{\partial x_p}\frac{e^{i\lambda_m|y-x|}}{|y-x|},\quad k,j=1,2,$$

$$\Psi^{(4)}_{kj}(y,x)=\sum_{l=1}^{4}(\delta_{kj}\gamma_l+\delta_l\frac{\partial^2}{\partial x_k\partial x_j})\frac{e^{i\lambda_m|y-x|}}{|y-x|},\quad k,j=1,2,$$

$r=|x-y|,$

$$\alpha_l=\frac{(-1)^l(\sigma_2^2-k_l^2)(\delta_{3l}+\delta_{4l})}{2\pi(\mu+\alpha)(k_3^2-k_4^2)},\quad \beta_l=-\frac{\delta_{1l}}{2\pi\rho\theta^2}+\frac{\alpha_l}{k_l^2},\ l=1,2,3,4,\ \sum_{l=1}^{4}\beta_l=0,$$

$$\gamma_l=\frac{(-1)^l(\sigma_1^2-k_l^2)(\delta_{3l}+\delta_{4l})}{2\pi(\beta+\nu)(k_3^2-k_4^2)},\quad \delta_l=-\frac{\delta_{2l}}{2\pi(j\theta^2-4\alpha)}+\frac{\gamma_l}{k_l^2},\quad l=1,2,3,4,\quad \sum_{l=1}^{4}\delta_l=0,$$

$$\varepsilon_l = \frac{(-1)^l(\delta_{3l}+\delta_{4l})}{2\pi(\beta+\nu)(k_3^2-k_4^2)}, \quad l=1,2,3,4, \ \sum_{l=1}^{4}\varepsilon_l = 0, \ \ k_1^2 = \frac{\rho\theta^2}{\lambda+2\mu}, \ k_2^2 = \frac{j\theta^2-4\alpha}{\varepsilon+2\nu},$$

$$k_3^2+k_4^2 = \sigma_1^2+\sigma_2^2+\frac{4\alpha^2}{(\mu+\alpha)(\beta+\nu)}, \ \ k_3^2k_4^2 = \sigma_1^2\sigma_2^2.$$

$T(\partial_y, n)$-kuchlanish operatori deyilib,

$$T(\partial_y,n) = T(\partial_y,n(y)) = \begin{Vmatrix} T^{(1)}(\partial_y,n) & T^{(2)}(\partial_y,n) \\ T^{(3)}(\partial_y,n) & T^{(4)}(\partial_y,n) \end{Vmatrix},$$

$$T^{(i)}(\partial_y,n) = \left\| T_{kj}^{(i)}(\partial_y,n) \right\|_{2\times 2}, \quad i=1,2,3,4,$$

$$T_{kj}^{(1)}(\partial_y,n) = \lambda n_k \frac{\partial}{\partial y_j} + (\mu-\alpha)n_j(y)\frac{\partial}{\partial y_k} + (\mu+\alpha)\delta_{kj}\frac{\partial}{\partial n(y)}, \ \ k,j=1,\,2,$$

$$T_{kj}^{(2)}(\partial_y,n) = T_{kj}^{(3)}(\partial_y,n) = 0, \ \ k,j=1,\,2,$$

$$T_{kj}^{(4)}(\partial_y,n) = \varepsilon n_k(y)\frac{\partial}{\partial y_j} + (\nu-\beta)n_j(y)\frac{\partial}{\partial y_k} + (\nu+\beta)\frac{\partial}{\partial n(y)}, \ \ k,j=1,\,2.$$

$n(y) = (n_1(y), n_2(y))$ – ∂D-chegaraning y nuqtasidagi tashqi normal vektori.

(2) formulani Somilian – Betti formulasi deb yuritamiz.

Karleman funksiyasini quramiz va u asosida qaralayotgan sistema uchun Karleman matritsasini quramiz

D - chegaralangan bir bog‘lamli soha, ∂D - uning chegarasi, $S-\partial D$ ning silliq qismi.

2-ta’rif. $y \neq x$ da aniqlangan, $\sigma > 0$ parametrdan bog‘liq, (4×4) o‘lchamli $\Pi_\sigma(y,x)$ matritsa $x \in D$ nuqtada va $\partial D \setminus S$ qism uchun Karleman matritsasi deyiladi, agar u quyidagi shartlarni qanoatlantirsa:

1) $\Pi_\sigma(y,x) = \Psi(y,x) + G_\sigma(y,x)$

bunda $G_\sigma(y,x) - (4\times 4)$ o‘lchamli matritsa y bo‘yicha (1) sistemani qanoatlantiradi,

2) $\forall x \in D$ da $\Pi_\sigma(y,x)$ matritsa ushbu

$$\int_{\partial D\setminus S}\left[\left|\Pi_\sigma(y,x)\right| + \left|T(\partial_y,n)\Pi_\sigma(y,x)\right|\right]dS_y \leq \alpha(\sigma),$$

tengsizlikni qanoatlantiradi, bu yerda $\alpha(\sigma)\to 0,\ \sigma\to\infty$.

D soha va $\partial D \setminus S$ qismi uchun Karleman matritsasi mavjud bo‘lsin. Bu holda quyidagi teorema o‘rinli

2-teorema. D da regulyar bo‘lgan (1) sistemaning yechimi quyidagi ko‘rinishga ega:

$$u(x) = \int_{\partial D} \left[\Pi_\sigma(y,x) \left\{ T(\partial_y, n) u(y) \right\} - u(y) \left\{ T(\partial_y, n) \Pi_\sigma(y,x) \right\} \right] \partial S_y, \; x \in D, \qquad (3)$$

bu yerda $\Pi_\sigma(y,x)$ - D soha uchun Karleman matritsasi.

Endi elastiklik nazariyasi sistemasining maxsus ko‘rinishdagi fundamental yechimlarini qurishga o‘tamiz. $K(w)$, $w = u + iv$(u, v - haqiqiy) – butun funksiya, quyidagi shartlarni qanoatlantiruvchi w da haqiqiy bo‘lsin:

$$K(u) \neq 0, \; \sup_{v \geq 1} \left| v^p K^{(\rho)}(u + iv) \right| \leq M(\rho, u) < \infty, \; \rho = 0,1,2,\ldots, \; u \in R^1$$

$\alpha = |y_1 - x_1|$ bo‘lsin.

$\Phi_\sigma(y,x,\lambda)$ funksiyani $\alpha > 0$ va $y \neq x$ larda quyidagi tenglik bilan aniqlaymiz:

$$2\pi K(\sigma x_2) \Phi_\sigma(y,x,\lambda) = \int_0^\infty \operatorname{Im} \frac{K(\sigma w)}{w - x_2} \cdot \frac{u J_0(\lambda u) du}{\sqrt{u^2 + \alpha^2}}, \qquad (4)$$

bu yerda $w = i\sqrt{u^2 + \alpha^2} + y_2$, $J_0(u)$-Besselning nolinchi tartibli funksiyasi.

[7] da quyidagi lemma isbotlanadi

Lemma.(4) formula bilan aniqlangan $\Phi_\sigma(y,x,\lambda)$ funksiya quyidagi ko‘rinishga ega:

$$\Phi_\sigma(y,x,\lambda) = \frac{\exp r}{2\pi r} + g_\sigma(y,x,\lambda)$$

bu yerda $g_\sigma(y,x,\lambda)$ - x, y ning barcha qiymatlarida aniqlangan va butun E^2 da y o‘zgaruvchi bo‘yicha Gelmgolts tenglamasining reguylyar yechimi, r = |x-y|.

$\Phi_\sigma(y,x,\lambda)$ funksiya yordamida quyidagi matritsani quramiz:

$$\Pi_\sigma(y,x) = \Pi(y,x,\sigma) = \left\| \begin{matrix} \Pi^{(1)}(y,x,\sigma) & \Pi^{(2)}(y,x,\sigma) \\ \Pi^{(3)}(y,x,\sigma) & \Pi^{(4)}(y,x,\sigma) \end{matrix} \right\|,$$

$$\Pi^{(i)}(y,x,\sigma) = \|\Pi_{kj}^{(i)}(y,x,\sigma)\|_{2\times 2}, \quad i = 1,2,3,4,$$

$$\Pi_{kj}^{(1)}(y,x,\sigma) = \sum_{m=1}^{4} \left(\delta_{kj} \alpha_m + \beta_m \frac{\partial^2}{\partial y_k \partial y_j} \right) \cdot \Phi_\sigma(y,x,i\lambda_m),$$

$$\Pi_{kj}^{(2)}(y,x,\sigma) = \Pi_{kj}^{(3)}(y,x,\sigma) = 0,$$

$$\Pi_{kj}^{(4)}(y,x,\sigma) = \sum_{m=1}^{4} \left(\delta_{kj} \gamma_m + \delta_m \frac{\partial^2}{\partial y_k \partial y_j} \right) \cdot \Phi_\sigma(y,x,i\lambda_m). \qquad (5)$$

3-teorema. (5) formula bilan aniqlangan $\Pi(y,x,\sigma)$ matritsa Karleman matritsasi bo‘ladi.

Isbot. Karleman matitsasining 1)-sharti lemmadan osongina kelib chiqadi, haqiqattan ham

$$\Pi(y,x,\sigma)=\Psi(y,x)+G(y,x,\sigma),$$

bunda

$$G(y,x,\sigma)=\begin{Vmatrix} G^{(1)}(y,x,\sigma) & G^{(2)}(y,x,\sigma) \\ G^{(3)}(y,x,\sigma) & G^{(4)}(y,x,\alpha\sigma) \end{Vmatrix},$$

$$G^{(i)}(y,x,\sigma)=\left\| G^{(i)}_{kj}(y,x,\sigma)\right\|_{2\times 2}, \quad i=1,2,3,4,$$

$$G^{(1)}_{kj}(y,x,\sigma)=\sum_{l=1}^{4}(\delta_{kj}\alpha_l+\beta_l\frac{\partial^2}{\partial x_k\partial x_j})g_\sigma(y,x,\lambda_l), \quad k,j=1,2,$$

$$G^{(2)}_{kj}(y,x,\lambda_l)=G^{(3)}_{kj}(y,x,\lambda_l)=0, \quad k,j=1,2,$$

$$G^{(4)}_{kj}(y,x,\lambda_l)==\sum_{l=1}^{4}(\delta_{kj}\gamma_l+\delta_l\frac{\partial^2}{\partial x_k\partial x_j})g_\sigma(y,x,\lambda_l), \quad k,j=1,2.$$

Karleman matitsasining 2)-sharti, (4) dan kelib chiqadi.

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