# MAXSUS SOHALARDA KARLEMAN MATRITSASI

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## ANNOTASIYA

Bu ishda momentli elastikliklik nazariyasi tenglamalari sistemasi yechimini fazoda yechim va uning kuchlanishi soha chegarasining musbat o‘lchovli qismida berilganda sohaning ichiga topish masalasi qaraladi. Bunday masalaga Koshi masalasi deyiladi. Qaralayotgan masalaning yechimi mavjudligi kriteriyasi keltiriladi.

**Kalit so‘zlar:** Momentli elastikliklik nazariyasi, Karleman funksiyasi, Karleman matritsasi, Koshi masalasi.

## KIRISH

Bu ish elastiklik nazariyasi tenglamalari sistemasi yechimini tekislikdagi chegaralanmagan sohada chegaraning qismida uning berilgan qiymatlari va uning kuchlanishi qiymatlari bo‘yicha davom ettirish masalasi, ya’ni elastiklik nazariyasi tenglamalari sistemasi uchun Koshi masalasi o‘rganiladi.

Bu ishda asosan chegaralanmagan sohada mos ravishda chegaralanmagan yechim holida Karleman matritsasini qurish yo‘llari o‘rganilgan. Karleman matritsasini qurishda bundan oldin qaralgan holdan farqli ravishda maxsuslik tartibi katta bo‘lgan yadro holida Karleman matritsasini mustaqil ravishda qurildi va shu holda regulyarlashgan yechim bilan aniq yechim orasidagi farq baholangan.

**Usullari.**

$$2\pi K(\sigma x_2)\Phi_\sigma(y,x,\lambda)=\int_0^\infty \operatorname{Im}\frac{K(\sigma w)}{w-x_2}\cdot\frac{uJ_0(\lambda u)du}{\sqrt{u^2+\alpha^2}}$$

va

$$\Pi_{kj}^{(4)}(y,x,\sigma)=\sum_{m=1}^{4}\left(\delta_{kj}\gamma_m+\delta_m\frac{\partial^2}{\partial y_k\partial y_j}\right)\cdot\Phi_\sigma(y,x,i\lambda_m)$$

formulalarda $K(w)$ funksiyani aniq ko‘rinishda tanlab, momentli elastiklik nazariyasi sistemasi uchun Karleman matritsasini maxsus sohada aniq ko‘rinishda quramiz.

1. $D$ elastik muhit $E^2$ da chegaralangan bir bog‘lamli, chegarasi $y_1 = 0$ haqiqiy o‘qning $l$ bo‘lagi va $y_2 > 0$ yarim tekislikda yotuvchi silliq $S$ egri chiziqdan iborat bo‘lsin, ya’ni $D$ qalpoq shaklidagi soha.

$\sigma > 0$ da (4) formulada

$K(w) = \exp \sigma w$, $w = i\sqrt{u^2 + \alpha^2}$

ifodani qo‘yamiz va quyidagiga ega bo‘lamiz:

$$2\pi \exp(\sigma x_2)\Phi_\sigma(y, x, \lambda) = \int_0^\infty \operatorname{Im} \frac{\exp(\sigma w)}{w - x_2} \cdot \frac{uJ_0(\lambda u)du}{\sqrt{u^2+\alpha^2}} \qquad (1)$$

(1) formuladan ko‘rinadiki, $l$ da $\Phi_\sigma(y, x, \lambda)$ funksiya, uning $\nabla\Phi_\sigma(y, x, \lambda)$ gradiyenti va uning ikkinchi tartibli xususiy xosilalari

$$\frac{\partial^2 \Phi_\sigma(y, x, \lambda)}{\partial y_i \partial y_j}, \quad i, j = 1,2,$$

$\sigma \to \infty$ da barcha $y$ va $x \in E^2$ larda eksponensial nolga intiladi.

(5) ga (6) ni qo‘yib, $\Pi(y, x, \sigma)$ hosil qilamiz.Quyidagi teorema o‘rinli

**4-teorema.** Qurilgan $\Pi(y, x, \sigma)$ matritsa $D$ soha va $l$ qism uchun Karleman matritsasidan iborat bo‘ladi.

**Isbot.** $\Pi_\sigma(y, x) = \Psi(y, x) + G_\sigma(y, x)$ ifoda va $G_\sigma(y, x)$ ning

$$\begin{cases} (\mu + \alpha)\Delta u + (\lambda + \mu - \alpha) grad div u + 2\alpha\, rot w + \rho\theta^2 u = 0, \\ (\nu + \beta)\Delta w + (\varepsilon + \nu - \beta) grad div w + 2\alpha\, rot u - 4\alpha w + j\theta^2 w = 0, \end{cases}$$

sistema yechimi ekanligi lemmadan birdan kelib chiqadi.

$$\int_l \left[ |\Pi(y, x, \sigma)| + |T(\partial_y, n)\Pi(y, x, \sigma)| \right] dS_y \le c(\lambda, \mu, x)\sigma \exp(-\sigma x_2) \qquad (2)$$

ekanligini isbotlaymiz.

$$\frac{\partial \Phi_\sigma(y, x)}{\partial y_1} = \frac{\exp \sigma(y_2 - x_2)\left[(y_2 - x_2)\sin \sigma|y_1 - x_1| + |y_1 - x_1|\cos \sigma(y_2 - x_2)\right]}{2\pi |x - y|^2},$$

$$\frac{\partial \Phi_\sigma(y, x)}{\partial y_2} = \frac{\exp \sigma(y_2 - x_2)\left[(y_2 - x_2)\cos \sigma|y_1 - x_1| + |y_1 - x_1|\sin \sigma(y_2 - x_2)\right]}{2\pi |x - y|^2}$$

va

$$|\Phi_\sigma(y, x)| \le \frac{const\left(\frac{1}{|x - y|}\right)}{\exp \sigma(y_2 - x_2)}$$

bo‘lgani uchun quyidagiga ega bo‘lamiz:

$$|\Pi(y,\lambda,\sigma)| \le \frac{c_1(\lambda,\mu)}{\ln\left(\frac{1}{|x-y|}\right)} \exp\sigma(y_2 - x_2)$$

$$|T(\partial,n)\Pi(y,x,\sigma)| \le \frac{c_2(\lambda,\mu)}{\left(\frac{\sigma}{|x-y|}\right)} \exp\sigma(y_2 - x_2)$$

Ushbu oxirgi tengsizliklardan (2) ni olamiz. Teorema isbotlandi.

Endi $D_p \subset E^2$ -chegaralangan bir bog‘lamli soha bo‘lib chegarasi quyidagi konusning yon sirti

$$\Sigma : \alpha_1 = \tau y_2,\ \alpha_1 = |y_1|,\ \tau = tg\frac{\pi}{2\rho},\quad y_2 > 0,\quad \rho > 1,$$

va konus ichida yotgan $S$ - silliq chiziqdan iborat bo‘lsin.Quyidagi belgilashlarni kiritamiz

$$\tilde{\beta} = \tau y_2 - \alpha_0,\quad \gamma = \tau x_2 - \alpha_0,\quad \alpha_0^2 = x_1^2,\ s = \tilde{\alpha}^2 = (y_1 - x_1)^2\ ,$$

$$\omega = i\tau\sqrt{u^2 + s} + \tilde{\beta},\qquad \omega_0 = i\tau\tilde{\alpha} + \tilde{\beta}.$$

Parametr $\sigma > 0$ da (5),(6) formulalar yordamida $\Pi(y,x,\sigma)$ ni quyidagicha qurib olamiz.

$$\mathrm{K}(\omega) = \mathrm{E}_\rho\left(\sigma(\omega - x_2)\right),\qquad \rho > 1.$$

deb, bunda $E_p(\omega)$ - Mittag-Lefler funksiyasini olamiz.

$$2\pi K(\sigma x_2)\Phi_\sigma(y,x,\lambda) = \int_0^\infty \mathrm{Im}\frac{K(\sigma w)}{w - x_2} \cdot \frac{uJ_0(\lambda u)du}{\sqrt{u^2+\alpha^2}}$$

formulada

$$\Phi(y,x,\lambda) = \Phi_\sigma(y - x,\lambda),\quad \lambda > 0$$

$$2\pi\Phi_\sigma(y - x,\lambda) = \int_0^\infty Im\left[\frac{E_p\left(\sigma(i\sqrt{u^2+s} + y_2 - x_2)\right)}{i\sqrt{u^2+s} + y_2 - x_2}\right]\frac{J_0(\lambda u)\,du}{\sqrt{u^2+s}} \qquad (3)$$

ni olamiz. (8) formula bilan aniqlangan funksiya uchun ham yuqorida keltirilgan lemma o‘rinli va bundan hosil bo‘lgan matritsa Karleman matritsasi ekanligini yuqorida 3,4-teoremalardagidek isbotlash mumkin.

Bizga bundan keyin $E_\rho(w)$ funksiyaning ayrim xossalari kerak bo‘ladi. Ularning ayrimlarini keltiramiz.

Mittag-Lefler turidagi funksiya deb, quyidagi darajali qator ko‘rinishida tasvirlanga butun funksiyani tushunamiz.

$$E_\rho(w) = \sum_{n=0}^\infty \Gamma^{-1}\left(1 + \frac{n}{\beta}\right)w^n,\quad w = u + iv,$$

bu yerda $\Gamma(s)$ – Eylerning Gamma funksiyasini ifodalaydi.

Mittag-Lefler turidagi funksiyasi $E_\rho(w)$ moduli bo‘yicha etarli katta kompleks $w$ larda ajoyib assimtotik ko‘rinishlarga ega.

$0<\beta\le\pi,\quad \varepsilon_0>0$ sonlari uchun quyidagi belgilashlarni kritamiz. $\gamma(\varepsilon_0,\beta_0)$ deb $\zeta$ kompleks tekisligida shunday konturni belgilaymizki u $arg(\zeta)$ ning o‘sishi yo‘nalishida harakatlanib, quyidagi qismlardan iborat:

1) $arg(\zeta)=-\beta_0,\quad |\zeta|\ge\varepsilon_0$ -nurdan

2) $|\zeta|=\varepsilon_0$ aylananing $-\beta_0\le arg(\zeta)\le\beta_0$ yoyidan

3) $arg(\zeta)=\beta_0,\quad |\zeta|\ge\varepsilon_0$ nurdan.

$0<\beta_0<\pi$ holida $\gamma(\varepsilon_0,\beta_0)$ kontur $\zeta$ tekislikni ikkita: $D^-$ va $D^+$ cheksiz sohalarga ajratadiki, bunda $D^-$ $\gamma(\varepsilon_0,\beta_0)$ dan chapda, $D^+$ esa o‘ngda.

$\rho>1,\quad \dfrac{\pi}{2\rho}<\beta_0<\dfrac{\pi}{\rho}$ uchun quyidagi belgilashni olamiz:

$$\Psi_\rho(w)=\frac{\rho}{2\pi i}\int\limits_{\gamma(\varepsilon_0,\beta_0)}\frac{\exp\zeta^\rho}{\zeta-w}d\zeta.$$

Bu holda quyidagi integral tasvir o‘rinlidir:

$$E_\rho(w)=\Psi_\rho(w),\qquad w\in D^-,$$

$$E_\rho(w)=\rho\exp w^\rho+\Psi_\rho(w),\qquad w\in D^+.$$

Bu formulalardan ayrim $\eta_0>0$ uchun quyidagi baholashlarni olishimiz mumkin.

$$|E_\rho(w)|\le\rho\exp(Rew^\rho)+|\Psi_\rho(w)|,\quad |argw|\le\frac{\pi}{2\rho}+\eta_0,$$

$$|E_\rho(w)|=|\Psi_\rho(w)|,\quad \frac{\pi}{2\rho}+\eta_0\le|argw|\le\pi,$$

$$|\Psi_\rho(w)|\le\frac{const}{1+|w|},\ E_\rho(w):\rho\exp w^\rho,\quad w>0,\quad w\to\infty.$$

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