RIMAN-LUIVILL KASR TARTIBLI INTEGRALI VA HOSILASIGA OID AYRIM MASALALARNING ISHLANISHI

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ANNOTATSIYA

Ushbu maqolada kasr tartibli xususiy hosilali differensial tenglamalarning eng ko‘p tarqalgan turlaridan bii bo‘lgan Riman- Liuvill integrali va hosilasiga oida ta’rif va tushunchalar keltirilgan. Shu bilan birgalikda integro-differensial operatorlarning bir necha ko‘rinishlari hamda ayrim integral va hosilalarning ishlanish usullari aniq hisoblangan.

Kalit so‘zlar: Kasr tartibli integrallar, matematik analiz, Riman integrali, kasr integro – differensial hisob, differensial operator, haqiqiy son.

KIRISH

So‘nggi yillarda ko‘pgina hayotiy jarayonlarning matematik modelini tuzib, uni matematik usullar bilan yechish matematiklar ichida keng tarqaldi. Bu jarayonlar meditsina va texnikalar rivojlanishi bilan uzviy bog‘liqdir. Kasr tartibli integral va hosilalarning fizika, biologiya, meditsina va texnika sohalariga tadbiqi juda ko‘p bo‘lib, u bu sohalarning rivojlanishida muhim ahamiyatga ega. Shu sababli so‘nggi yillarda matematiklar orasida kasr tartibli hosila qatnashgan differensial va xususiy hosilali differensial tenglamalarni o‘rganishga bo‘lgan qiziqish ortib bormoqda. Kasr tartibli tenglamalar bir vaqtda diffuziya va to‘lqin tarqalish jarayonlarini ifodalaydi. Bunday jarayonlar tabiatda juda ko‘p uchraydi. Shuning uchun ham kasr tartibli tenglamalarni o‘rganish juda muhim ahamiyatga egadir. Kasr tartibli hosilali va integrallarning eng ko‘p tarqalgan turlaridan ikkitasi bu Riman - Luivill va Caputo kasr tartibli hosilalalari hisoblanadi.

Ushbu maqolada Riman-Luivill ma’nosidagi kasr tartibli hosila va kasr tartibli integrallar bilan tanishamiz. Riman-Luivill integrali va hosilasi Bernhard Riman va Jozef Liouville sharafiga nomlangan. 1832 yilda birinchi bora kasr tartibli hosila va integrallarni hisoblash ko‘rib chiqilgan.

Kasr tartibli hosilalar va integrallar tushunchasi kiritilishi bilan hosila integral orasidagi chegara keskin yo‘qolib ketadi. Integral – differentsial operatorlarning kasr tartibli operatorlariga Koshi tipidagi umumlashgan integral formulasi kasr tartibli differentegrallarining quyidagi ta'riflariga olib keladi:

$$(I_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)}{(x-t)^{1-\alpha}} dt, \qquad (1)$$

$$(D_{a+}^{\alpha} f)(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_a^x \frac{f(t)}{(x-t)^{\alpha-n+1}} dt, \qquad (2)$$

bu yerda: $\alpha \in \mathbb{R}$, $n-1<\alpha<n$

$I_{a+}^{\alpha} - \alpha$ tartibli integral operator.

$D_{a+}^{\alpha} - \alpha$ tartibli differensial operator.

Kasr tartibli hosilali va integralli operatorning bu ta'rifi yagona emas. Integrodifferensial operatorlarning Veyl, Grunwald-Letnikov, Caputo va boshqa ko'rinishlari ham ma'lumdir.

Riman-Luivill integrali va hosilasi.

$$I^{\alpha} f = \frac{1}{\Gamma(\alpha)} \int_0^x f(t)(x-t)^{\alpha-1} dt. \qquad (3)$$

Endi $D^{\alpha} = \frac{d^{\alpha}}{dx^{\alpha}}$, ni aniqlaymiz.

Biz D^{α} ni I^{α} operatorning chap teskari operatori sifatida aniqlaymiz.

1-tasdiq. $D^{\alpha} I^{\alpha} = J$ ayniyatga ko'ra $0<\alpha<1$ bo'lgan hol uchun $D^{\alpha} = DI^{1-\alpha}$

tenglik kasr tartibli differensial operator bo'ladi.

Bu tasdiqni isbotlashdan avval $\forall \alpha, \beta > 0$ sonlar uchun quyidagi tenglikni o'rinli ekanligidan foydalanamiz. $I^{\alpha} I^{\beta} = I^{\alpha+\beta} = I^{\beta} I^{\alpha}$ (kommuntativlik), (3) formulaga ko'ra isbot qilinadi.

D^{α} ni I^{α} operatorning chap teskari operatori ekan demak $D^{\alpha} I^{\alpha} = J$ ayniyat o'rinli. D^{α} ning tasdiqda keltirilgan ifodasidan foydalanib ayniyatni tekshirib ko'raylik:

$$D^{\alpha} I^{\alpha} = DI^{1-\alpha} I^{\alpha} = [kommuntativlik] = DI^{1-\alpha+\alpha} = DI = J$$

Tasdiq o'rinli bo'ldi.

Endi hozirgina isbotlangan tasdiq hamda (3) formula yordamida $0<\alpha<1$ bo'lgan hol uchun kasr tartibli hosila tushunchasini kiritamiz:

$$D^{\alpha} f(x) = DI^{1-\alpha} f(x) = \frac{d}{dx} \frac{1}{\Gamma(1-\alpha)} \int_0^x f(t)(x-t)^{1-\alpha-1} dt.$$

$$=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_0^x\frac{f(t)}{(x-t)^{\alpha}}dt. \qquad (4)$$

$D^{\alpha}=D^{n}I^{n-\alpha}$ - bu operator tengliklarni analitik formula ko‘rinishida tasvirlaymiz:

$$D^{\alpha}f=D^{n}I^{n-\alpha}=\left(\frac{d}{dx}\right)^{n}\frac{1}{\Gamma(n-\alpha)}\int_0^x\frac{f(t)}{(x-t)^{\alpha-n+1}}dt.$$

Bu formula tartibi ixtiyoriy nomanfiy haqiqiy son bo‘lgan kasr tartibli hosila uchun Riman-Luivill formulasi deyiladi. Ixchamlab yozsak quyidagini olamiz:

$$D^{\alpha}f=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_0^x\frac{f(t)}{(x-t)^{\alpha-n+1}}dt. \qquad (5)$$

Masalan: 1) $f(t)=t^{\mu}$ funksiyaning Riman-Luivill integrali hisoblang.

$$I^{\alpha}f(\mathrm{t})=\frac{1}{\Gamma(\alpha)}\int_0^t f(\xi)(t-\xi)^{\alpha-1}d\xi=\frac{1}{\Gamma(\alpha)}\int_0^t\xi^{\mu}(t-\xi)^{\alpha-1}d\xi=$$

$$=\begin{bmatrix}\xi=ty\\ d\xi=tdy\end{bmatrix}=\frac{1}{\Gamma(\alpha)}\int_0^1 t^{\mu}y^{\mu}(t-ty)^{\alpha-1}tdy=\frac{t^{\mu+\alpha}}{\Gamma(\alpha)}\int_0^1 y^{\mu}\cdot(1-y)^{\alpha-1}dy=$$

$$=\frac{t^{\mu+\alpha}}{\Gamma(\alpha)}\cdot B(\alpha,\mu+1)=\frac{t^{\mu+\alpha}}{\Gamma(\alpha)}\cdot\frac{\Gamma(\alpha)\cdot\Gamma(\mu+1)}{\Gamma(\alpha+\mu+1)}=\frac{\Gamma(\mu+1)}{\Gamma(\alpha+\mu+1)}t^{\mu+\alpha}$$

$$I^{\alpha}(t^{\mu})=\frac{\Gamma(\mu+1)}{\Gamma(\alpha+\mu+1)}t^{\mu+\alpha} \qquad (6)$$

(1.3.6) formulaga keltirib qo‘yib quyidagi integralni hisoblaymiz.

2) $I^{\frac{1}{2}}(t^{\frac{1}{2}})=\dfrac{\Gamma(\frac{1}{2}+1)}{\Gamma(\frac{1}{2}+\frac{1}{2}+1)}t^{\frac{1}{2}+\frac{1}{2}}=\dfrac{\Gamma(\frac{3}{2})}{\Gamma(2)}t=\Gamma(1+\frac{1}{2})t=\frac{1}{2}\Gamma(\frac{1}{2})\mathrm{t}=\dfrac{t}{2\sqrt{\pi}}$.

3) $f(t)=\sin t$ funksiyaning Riman-Luivill integrali hisoblang.

Yuqoridagi funksiyaning integralini ham (1.3.5) integralga o‘xshab hisoblasak quyidagi tenglik hosil bo‘ladi.

$$I^{\alpha}(\mathrm{sint})=t^{\alpha}\,\mathrm{E}_{2,\alpha+2}(-t^{2}) \qquad (7)$$

$$4) I^{\alpha}(e^{t}) = \frac{1}{\Gamma(\alpha)}\int_0^t f(\xi)(t-\xi)^{\alpha-1}d\xi = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\xi)^{\alpha-1}e^{\xi}d\xi =$$

$$= \frac{1}{\Gamma(\alpha)}\int_0^t (t-\xi)^{\alpha-1}\cdot\sum_{k=0}^{\infty}\frac{\xi^k}{k!}d\xi = \frac{1}{\Gamma(\alpha)}\sum_{k=0}^{\infty}\frac{1}{k!}\cdot\int_0^t (t-\xi)^{\alpha-1}\xi^k =$$

$$= \frac{1}{\Gamma(\alpha)}\sum_{k=0}^{\infty}\frac{1}{k!}\cdot\int_0^1 (t-ty)^{\alpha-1}t^k y^k tdy\begin{bmatrix}\xi = ty\\ d\xi = tdy\end{bmatrix} =$$

$$= \frac{1}{\Gamma(\alpha)}\sum_{k=0}^{\infty}\frac{t^{\alpha+k}}{k!}\int_0^1 (1-y)^{\alpha-1}y^k tdy = \frac{1}{\Gamma(\alpha)}\sum_{k=0}^{\infty}\frac{t^{\alpha+k}}{k!}\cdot B(\mathrm{k}+1,\alpha) =$$

$$= \frac{1}{\Gamma(\alpha)}\cdot\frac{\Gamma(\alpha)\Gamma(k+1)}{\Gamma(\alpha+k+1)}\cdot\sum_{k=0}^{\infty}\frac{t^{\alpha+k}}{k!} = \frac{\Gamma(k+1)}{\Gamma(\alpha+k+1)}\cdot\sum_{k=0}^{\infty}\frac{t^{\alpha+k}}{k!} =$$

$$= \frac{\Gamma(k+1)}{\Gamma(\alpha+k+1)}\cdot\frac{1}{\Gamma(k+1)}\cdot\sum_{k=0}^{\infty}t^{\alpha+k} = \sum_{k=0}^{\infty}\frac{t^{\alpha+k}}{\Gamma(\alpha+k+1)} =$$

$$= t^{\alpha}\sum_{k=0}^{\infty}\frac{t^k}{\Gamma(k+\alpha+1)} = t^{\alpha}\cdot E_{1,\alpha+1}(\mathrm{t})$$

$$I^{\alpha}(e^{t}) = t^{\alpha}\cdot E_{1,\alpha+1}(\mathrm{t}) \tag{8}$$

5) $\mathrm{f(t)} = \mathrm{C}$ o‘zgarmas sonning $\alpha = \frac{1}{2}$ bo‘lganda Riman-Luivill hosilasini hisoblang.

$$D^{\frac{1}{2}}y(t) = \frac{1}{\Gamma(1-\frac{1}{2})}\frac{d}{dt}\int_0^t \frac{C}{(\mathrm{t}-\xi)^{\frac{1}{2}}}d\xi = \frac{C}{\Gamma(\frac{1}{2})}\frac{d}{dt}\int_0^t (t-\xi)^{(-\frac{1}{2})}d\xi =$$

$$= \frac{C}{\Gamma(\frac{1}{2})}\frac{d}{dt}\left(-2(t-\xi)^{\frac{1}{2}}\Big|_0^t\right) = \frac{C}{\Gamma(\frac{1}{2})}\frac{d}{dt}(2t^{\frac{1}{2}}) = \frac{C}{\Gamma(\frac{1}{2})}t^{(-\frac{1}{2})} = \frac{C}{\sqrt{\pi t}}.$$

Demak, $D^{\frac{1}{2}}y(t) = \frac{C}{\sqrt{\pi t}} \Rightarrow D^{\frac{1}{2}}[5] = \frac{5}{\sqrt{\pi t}}$

6) $f(t) = t^{\mu}$ funksiyaning $n-1 \prec \alpha \prec n$ bo‘lganda Riman-Luivill hosilasini hisoblang.

$$D^{\alpha}f(t) = \frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}\int_0^t \frac{\xi^{\mu}}{(t-\xi)^{\alpha-n+1}}d\xi =$$

$$=\frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}\int_0^t (t-\xi)^{-\alpha+n-1}\xi^{\mu}d\xi\left[\xi=ty\right]=$$

$$=\frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}\int_0^1 (t-ty)^{-\alpha+n-1}t^{\mu}y^{\mu}tdy=$$

$$=\frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}\int_0^1 (1-y)^{-\alpha+n-1}y^{\mu}t^{\mu-\alpha+n}dy=\frac{B(\mu+1,\mathrm{n}-\alpha)}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}(t^{\mu-\alpha+n})=$$

$$=\frac{\Gamma(\mu+1)}{\Gamma(\mu+n-\alpha+1)}\frac{d^n}{dt^n}(t^{\mu-\alpha+n})=\frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)}(t^{\mu-\alpha})$$

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