STIFEL KO‘PXILLIGI GEOMETRIYASI

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ANNOTATSIYA

Ushbu maqolada Stifel ko‘pxilligi evklid fazosiga akslantirishda sirt o‘lchami keltirilgan. $V(n,k)$ to‘plam Stiefel ko‘pxilligi ekanligi ko‘rsatilgan.

Kalit so‘zlar: Stifel ko‘pxilligi, ko‘pxilliklar, izometrik akslantirish, nuqta, aylana, sfera.

Biz $V(n,k)$ bilan k o‘lchovli evklid fazosini n o‘lchovli fazoga chiziqli izometrik akslantirish to‘plamini belgilaylik. Bu yerda $k, n \in Z^+ \cup 0$ hamda

$0 \le k \le n$ shartni qanotlantiruvchi butun sonlar. Bu to‘plamning ko‘pxillik ekanligini ko‘rsatamiz va uni Shtifell ko‘pxilligi deb ataymiz.

Biz k o‘lchovli evklid fazosini n o‘lchovli evklid fazosiga akslantitishni aniqlash uchun fazoda birorta ortonormal bazis tanlaymiz

$$\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_k \qquad (e_i * e_j) = \delta_{ij} = \begin{cases} 1 \, agar \; i = j \; bo'lsa, \\ 0 \, agar \; i \ne j \; bo'lsa \end{cases}$$

Shunda $\bar{x} \in R^k$

$$\bar{x} = x^1 \vec{e}_1 + x^2 \vec{e}_2 + \ldots + x^k \vec{e}_k$$

ko‘rinishda ifodalanadi.

Chiziqli izometrik akslantirish ortogonal matrisa yordamida berilganli uchun uni

$$f\left(\bar{x}\right) = x^1 A\vec{e}_1 + x^2 A\vec{e}_2 + \ldots + x^k A\vec{e}_k$$

ko‘rinishda yozamiz. A ortoganal matritsa bo‘lgani uchun

$$A * A^T = E$$

Biz matritsani

$$A = \begin{pmatrix} & & & \\ v_{11} & v_{21} & \cdots & v_{k1} \\ v_{12} & v_{22} & \cdots & v_{k2} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1n} & v_{2n} & \cdots & v_{kn} \end{pmatrix}$$

Korinishda yozamiz. A matritsaning elementlari soni $n*k$ ta elementdan iborat.

$$\left(A\vec{e_i}; A\vec{e_j}\right) = \begin{cases} 1 \text{ agar } i = j \text{ bo'lsa} \\ 0 \text{ agar } i \neq j \text{ bo'lsa} \end{cases}$$

Demak, $i \neq j$ holda tenglamalar soni qaraymiz $\left(A\vec{e_1}; A\vec{e_j}\right)$ bo‘lganda 0 ga teng bo‘lgan tenglamalar $k-1$ ta ; $\left(A\vec{e_2}; A\vec{e_j}\right)$ bo‘lganda esa $k-2$ ta buni davom ettirib $\left(A\vec{e_{k-1}}; A\vec{e_j}\right)$ bo‘lganda 1 ta ularning hammasi

$$k-1+k-2+\ldots+2+1 = \frac{k-1+1}{2}(k-1) = \frac{k(k-1)}{2} \text{ ta}$$

Hamda $i = j$ holda 1 ga teng bo‘lgan tenglamalar soni $\left(A\vec{e_1}; A\vec{e_1}\right)$ 1 ta ; $\left(A\vec{e_2}; A\vec{e_2}\right)$ 1 ta ; $\left(A\vec{e_3}; A\vec{e_3}\right)$ 1 ta ; ... ; $\left(A\vec{e_k}; A\vec{e_k}\right)$ 1 ta bu hollarda hammasi bo‘lib k ta tenglama. Biz barcha $i \neq j$ va $i = j$ hollardagi tenglamalarni sonini aniqlashimiz uchun yuqoridagi hollarda hosil bo‘lgan barcha tenglamalar sonini qo‘shamiz. Natijada

$$k + \frac{k(k-1)}{2} = \frac{k(k-1+2)}{2} = \frac{k(k+1)}{2} \text{ ni hosil qilamiz.}$$

Bu akslantirishda sirt o‘lchami quyidagicha kamayadi

$$nk - \frac{k(k+1)}{2} \qquad (1)$$

(1) Formula ixtiyoriy $V(n,k)$ fazoda Stifel ko‘pxilligini $f(\overline{x})$ akslantirishdagi sirtini o‘lchamini ifodalaydi.

Umumiy xolda aytganda $V(n,0)$ holda bu to‘plam $f(\overline{x})$ akslantirishimizda S^0 ya’ni nuqtaga akslanar ekan. Haqiqatan ham, $V(n,0)$ –nol o‘lchovli evklid fazosini n o‘lchovli fazoga chiziqli akslantiruvchi izometrik akslantirish to‘plami.

$V(n,1) - S^{n-1}$ sferaga akslanadi. $V(n,2), V(n,3), \ldots, V(n,n)$ hollarda hosil bo‘ladigan tenglamalar sonini $V(n,k)$ to‘plam elementlari sonidan ayirsak yuqoridagi (1) formula kelib chiqishini ham ko‘rishimiz mumkin.

Misol uchun $V(2,1)$ to‘plam f akslantirishimizda $S^{2-1} = S^1$ yani aylanaga akslanar ekan haqiqatan ham, $V(2,1)$ –bir o‘lchovli evklid fazosini ikki o‘lchovli fazoga chiziqli akslantiruvchi izometrik akslantirish to‘plami.

$V(2,1)$ $R^1 \to R^2$ R^1 ga tegishli e birlik vektorni, hamda unga tegishli biror $\overline{x}$ nuqtani olamiz $\overline{x} = A\,\overline{x} = \lambda\vec{e}$ u holda $f(x) = \lambda A\vec{e}$ bo‘ladi. Bundan

$$\left(A\vec{e_1}; A\vec{e_1}\right) = \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix}\begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = v_{11}^2 + v_{12}^2 = 1$$

$v_{11}^2+v_{12}^2=1$ (2)

aylana tenglamasi hosil bo‘ldi. Biz buni o‘lchovi S^1 ga akslanishini aytgan edik. Endi buni (1) Shtifel fo‘rmulasiga qo‘yamiz: Bu yerda $n=2,\ k=1 \quad 2*1-\frac{1(1+1)}{2}=2-1=1$ $\mathrm{V}(2,1)$ to‘plamimiz 2 ta elementdan iborat, undan biz hosil qilgan yuqoridagi (2) bitta tenglama sonini ayirsak 2-1=1 bo‘lishini osongina ko‘ramiz.

Endi $\mathrm{V}(3,1)$ va $\mathrm{V}(3,2)$ hollarni ko‘raylik qolgan barcha hollar $\mathrm{V}(\mathrm{n},\mathrm{k})$ hammasi shu tartibda bajariladi.

$\mathrm{V}(3,1)$ $R^1 \to R^3$ R^1 dan e birlik vektor va biror $\bar{x}$ nuqta tanlab olamiz $\bar{x}=\lambda\vec{e}$ $f\left(\bar{x}\right)=\lambda A\vec{e}$ bu holda

$$\left(A\vec{e}_1;A\vec{e}_1\right)=\begin{pmatrix}v_{11}\\v_{12}\\v_{13}\end{pmatrix}\begin{pmatrix}v_{11}\\v_{12}\\v_{13}\end{pmatrix}=v_{11}^2+v_{12}^2+v_{13}^2=1 \text{ bo‘ladi}$$

Ya’ni $v_{11}^2+v_{12}^2+v_{13}^2=1$ (S^2) sfera tenglamasini ifodalaydi.

Biz (1) fo‘rmuladan ham $n=3,\ k=1$ dan $3*1-\frac{1(1+1)}{2}=3-1=2$ S^2 ekanini ko‘rsata olamiz. Keyingi barcha $\mathrm{V}(\mathrm{n},1)$ hollarda – S^{n-1} sferaga akslanishi ravshan.

$\mathrm{V}(3,2)$ holatni ham ko‘rsak bunda f funksiya $R^2 \to R^3$ bajaradi. Endi R^2 ga tegishli ikkita e_1,e_2 vector hamda biror $\bar{x}=\lambda^1\vec{e_1}+\lambda^2\vec{e_2}$ nuqta olamiz. U holda f akslantiruvchi funksiya $f(\bar{x})=\lambda^1A\vec{e_1}+\lambda^2A\vec{e_2}$ bo‘ladi.

$$\left(A\vec{e}_1;A\vec{e}_1\right)=\begin{pmatrix}v_{11}\\v_{12}\\v_{13}\end{pmatrix}\begin{pmatrix}v_{11}\\v_{12}\\v_{13}\end{pmatrix}=v_{11}^2+v_{12}^2+v_{13}^2=1$$

va

$$\left(A\vec{e}_2;A\vec{e}_2\right)=\begin{pmatrix}v_{21}\\v_{22}\\v_{23}\end{pmatrix}\begin{pmatrix}v_{21}\\v_{22}\\v_{23}\end{pmatrix}=v_{21}^2+v_{22}^2+v_{23}^2=1$$

bundan tashqari yana bitta tenglamamiz bor $\delta_{ij}=0$ holda

$$\left(A\vec{e}_1;A\vec{e}_2\right)=\begin{pmatrix}v_{11}\\v_{12}\\v_{13}\end{pmatrix}\begin{pmatrix}v_{21}\\v_{22}\\v_{23}\end{pmatrix}=v_{11}v_{21}+v_{12}v_{22}+v_{13}v_{23}=0$$

ularni sistema qilib yozsak $\begin{cases}v_{11}^2+v_{12}^2+v_{13}^2=1\\v_{21}^2+v_{22}^2+v_{23}^2=1\\v_{11}v_{21}+v_{12}v_{22}+v_{13}v_{23}=0\end{cases}$

3 ta tenglama hosil bo‘ldi $\mathrm{V}(3,2)$ to‘plam elementlari soni $3*2=6$ ta undan hosil qilgan tenglamalarimiz sonini ayirsak $6-3=3$ qoladi. Demak, n va k larni o‘zgartirib yuqoridagi (1) fo‘rmula har doim o‘rinli bo‘lishini ko‘rishimiz mumkin.

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