INTEGRAL VA UNING TATBIQLARINI O‘RGATISH METODIKASI

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ANNOTATSIYA

Mazkur maqolada aniqmas integralni hisoblashni o‘rgatish masala qo‘yilgan. Bunda dastlab, funksiyaning boshlang‘ich funksiyasi, aniqmas integral ta’rifi, aniqmas integralning xossalari va aniqmas integralning hisoblashga doir misollar yechishga doir ko‘rsatmalar berilgan.

Kalit so‘zlar: boshlang‘ich funksiya, o‘zgarmas son, aniqmas integral, integral belgisi, o‘zgaruvchini almashtirish.

Biz hozirgacha biror $y = f(x)$ funksiyasi berilgan bo‘lsa, bu funksiyaning hosilasini yoki differensialini hisoblashni o‘rgandik. Endi hosila olish amaliga teskari bo‘lgan amal tushunchasini kiritishga harakat qilamiz. Agar bizga hosilasi olingan funksiya berilgan bo‘lsa, ana shu fimksiyani hosilasi olingunga qadar, ya’ni uning boshlang‘ich ko‘rinishi qanday bo‘lgan edi degan savolga javob beramiz. Ta’rif. Agar $y = F(x)$ funksiyasining hosilasi $f(x)$ ga teng bo‘lsa, ya’ni $F(x) = f(x)$ tenglik o‘rinli bo‘lsa, u holda $F(x)$ funksiyasi $f(x)$ funksiya uchun boshlang‘ich funksiya deyiladi.

1-misol. Agar $f(x) = x^2$ bo‘lsa, uning boshlang‘ich funksiyasi $F(x) = \frac{x^3}{3}$ bo‘ladi, chunki $F'(x) = \frac{3x^2}{3} = x^2 = f(x)$ bo‘ladi.

2-misol. Agar $f(x) = \sin x$ bo‘lsa, uning boshlang‘ich funksiyasi $F(x) = -\cos x$ bo‘ladi, chunki $F'(x) = (-\cos x)' = \sin x = f(x)$ bo‘ladi.

3-misol. Agar $f(x) = \frac{1}{\sqrt{1-x^2}}$ bo‘lsa, uning boshlang‘ich funksiyasi uning boshlang‘ich funksiyasi $F(x) = \arcsin x$ bo‘ladi.

Yuqoridagi misollardan ko‘rinadiki, agar $f(x)$ funksiya uchun $F(x)$ funksiyasi boshlang‘ich funksiya bo‘ladigan bo‘lsa, u holda $F(x) + C$ funksiya ham boshlang‘ich funksiya bo‘ladi, chunki $[F(x) + C]' = f(x)$, C-o‘zgarmas son. Bundan ko‘rinadiki, agar $f(x)$ funksiyaning boshlang‘ich funksiyasi mavjud bunday boshlang‘ich funksiyasi cheksiz ko‘p bo‘lib, ular o‘zgarmas son C ga farq qilar ekan. 1-misolda $\frac{x^3}{3} + C$, 2-misolda $(-\cos x + C)$, 3-misolda $(\arcsin x + C)$ boshlang‘ich funksiyalar bo‘ladi.

Ta'rif. $f(x)$ funksiyaning boshlang'ich funksiyasining umumiy ko'rinishi $F(x) + C$ ga shu $f(x)$ funksiyaning aniqmas integrali deyiladi va uquyidagicha yoziladi:

$\int f(x)dx = F(x) + C.$

Bunda $\int$–integral belgisi, $f(x)dx$ integral ostidagi ifoda deb yuritiladi.

Ta'rif. $f(x)$ funksiyaning boshlang'ich funksiyasining umumiy ko'rinishi $F(x) + C$ ni topishamaliga integrallash amali deyiladi. Bu ta'rifdan ko'rinadiki, $f(x)$-funksiyaning integrallash amali shu funksiyaning hosila olish yoki differensiallash amalaiga nisbatan teskari bo'lgan amal ekan. Integrallash amali quyidagi muhim xossalarga ega:

1-xossa. Agar differensiallash belgisi integrallash belgisidan oldin kelsa, ular o'zaro teskari amallar bo'lgani uchun bir-birini yo'qotadi:

$$d\int f(x)dx = f(x)dx.$$

2-xossa. Differensial belgisi integral belgisidan keyinda kelsa, bu belgilar bir-birini yo'qotgandan so'ng $F(x)$ ga o'zgarmas C soni qo'shiladi:

$$\int df(x)dx = F(x) + C.$$

Isboti: $\int dF(x) = \int F(x)dx = \int f(x)dx = F(x) + C.$

3-xossa. O'zgarmas sonni integral ishorasi tashqarisiga chiqarib yozish mumkin:

$$\int kf(x)dx = k \cdot \int f(x)dx.$$

4-xossa. Algebraik yig'indi(ayirma)ning integrali qo'shiluvchi(ayriluvchi) integrallarining algebraik yig'indisiga(ayirmasiga) teng:

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx.$$

Integral jadvali

1. $\int dx = x + C.$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C.$
3. $\int \frac{dx}{x} = \ln|x| + C.$
4. $\int a^x\, dx = \frac{a^x}{\ln a} + C.$
5. $\int e^x\, dx = e^x + C.$
6. $\int \sin x dx = -\cos x + C.$
7. $\int \cos x dx = \sin x + C.$
8. $\int \frac{dx}{\cos^2 x} = tgx + C.$
9. $\int \frac{dx}{\sin^2 x} = -ctgx + C.$
10. $\int \frac{dx}{\sqrt{a^2+x^2}} = \arcsin\frac{x}{a} + C.$
11. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C.$
12. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + C.$
13. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C.$

Aniqmas integralda o‘zgaruvchini almashtirish.

Faraz qilaylik, $I = \int f(x)dx$ integralni hisoblash kerak bo‘lsin. Integral ostida shunday f(x) funksiyalar mavjud bo‘ladiki, bu funksiyalarning integralini hisoblash uchun yangi o‘zgaruvchi kiritishga to‘g‘ri keladi. Faraz qilaylik, $I = \int f(x)dx$ integralda $x = \varphi(t)$ o‘zgaruvchini almashtiraylik, u holda $dx = \varphi'(t)dt$ bo‘ladi. Ularni integral ostidagi ifodaga qo‘ysak, $\int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt$ bo‘ladi. Bu formula aniqmas integralda o‘zgaruvchini almashtirish formulasi deyiladi.

1-misol. $I = \int \frac{dx}{5-3x}$ ni hisoblang.

$$5 - 3x = t; \quad x = \frac{5-t}{3}; \quad dx = -\frac{1}{3}dt.$$

$$I = \int \frac{dx}{5-3x} = -\frac{1}{3}\int \frac{dt}{t} = -\frac{1}{3}\ln|t| = -\frac{1}{3}\ln|5-3x| + C.$$

2-misol. $I = \int \frac{dx}{1+\sqrt[3]{x+1}}$ ni hisoblang. Buni hisoblash uchun o‘zgaruvchini almashtirish usulidan foydalanamiz.

$t = \sqrt[3]{x+1};\ t^3 = x + 1;\ x = t^3 - 1; \quad dx = 3t^2dt.$

$I = \int \frac{dx}{1+\sqrt[3]{x+1}} = \int \frac{3t^2dt}{1+t} = 3\int \frac{t^2-1+1}{1+t}dt = 3\left(\int \frac{t^2-1}{1+t}dt + \int \frac{dt}{1+t}\right) =$

$= 3\left(\frac{(t-1)(t+1}{t+1}dt + \int \frac{dt}{1+t}\right) = 3\left(\frac{t^2}{2} - t + \ln|1+t| + C\right) =$

$= -\frac{3\sqrt[3]{(x+1)^2}}{2} - 3\sqrt[3]{x+1} + 3\ln\left|1 + \sqrt{x+1}\right| + C.$

3-misol. $I = \int 3^{5x}dx$ integralni hisoblang.

$$I = \int 3^{5x}dx = \frac{3^{5x}}{5\ln 3} + C.$$

4-misol. $I = \int 19^{2x}dx$ integralni hisoblang.

$$I = \int 19^{2x}dx = \frac{19^{2x}}{2\ln 19} + C.$$

5-misol. $I = \int 7^{2x}dx$ integralni hisoblang.

$$I = \int 7^{2x}dx = \frac{7^{2x}}{2\ln 7} + C.$$

6-misol.Quyidagi integrallarni hisoblang:

a) $\int \sin 3x dx$ b) $\int \sin 4x dx$ v) $\int \sin 33x dx$.

Bilishimiz lozim: $y' = (\cos x)' = -\sin x$, $y' = (\cos 2x)' = -2\sin 2x$

$$\int \sin x dx = -\cos x + C.$$

a) $\int \sin 3x dx = -\frac{\cos 3x}{3} + C.$

b) $\int \sin 4xdx = -\frac{\cos 4x}{4} + C.$

v $\int \sin 33xdx = -\frac{\cos 33x}{33} + C.$

7-misol. Quyidagi integrallarni hisoblang: a) $\int \sin 3x\cos 2xdx$

b) $\int \sin 5x\cos 3xdx$ v) $\int \cos 2x\cos 3xdx$ g) $\int \sin 7x \sin 3xdx$

d) $\int \sin 2x\cos xdx.$

Quyidagi trigonometrik formulalarni bilishimiz lozim:

1) $\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y)).$

2) 1) $\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y)).$

3) 1) $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y)).$

$$a) \int \sin 3x\cos 2xdx = \frac{1}{2}\int(\sin 5x + \sin x)dx = \frac{1}{2}\left(\int \sin 5xdx + \int \sin xdx\right) =$$
$$= \frac{1}{2}\frac{1}{5}(-\cos 5x) + \frac{1}{2}(-\cos x) + C = -\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C.$$

$$b) \int \sin 5x\cos 3xdx = \frac{1}{2}\int(\sin 8x + \sin 2x)dx = \frac{1}{2}\left(\int \sin 8xdx + \int \sin 2xdx\right) =$$
$$= \frac{1}{2}\frac{1}{8}(-\cos 8x) + \frac{1}{2}\frac{1}{2}(-\cos 2x) + C = -\frac{1}{16}\cos 8x - \frac{1}{4}\cos 2x + C.$$
$$\frac{1}{2}\int(\sin 5x + \sin x)dx = \frac{1}{2}\left(\int \sin 5xdx + \int \sin xdx\right) =$$

$$v) \int \cos 2x\cos 3xdx = \frac{1}{2}\int(\cos 5x + \cos(-x))dx = \frac{1}{2}\left(\int \cos 5xdx + \int \cos xdx\right) =$$
$$= \frac{1}{2}\frac{1}{5}(\sin 5x) + \frac{1}{2}\sin x + C = \frac{1}{10}\sin 5x + \frac{1}{2}\sin x + C.$$

$$g) \int \sin 7x\sin 3xdx = \frac{1}{2}\int(\cos 4x + \cos 10x)dx = \frac{1}{2}\left(\int \cos 4xdx + \int \cos 10xdx\right) =$$
$$= \frac{1}{2}\frac{1}{4}\sin 4x + \frac{1}{2}\frac{1}{10}\sin 10x + C = \frac{1}{8}\sin 4x + \frac{1}{20}\sin 10x + C.$$

$$d) \int \sin 2x\cos xdx = \frac{1}{2}\int(\cos x + \cos 3x)dx = \frac{1}{2}\left(\int \cos xdx + \int \cos 3xdx\right) =$$
$$= \frac{1}{2}\sin x + \frac{1}{2}\frac{1}{3}\sin 3x + C = \frac{1}{2}\sin x + \frac{1}{6}\sin 3x + C.$$

8-misol. Quyidagi integrallarni hisoblang: a) $\int \frac{dx}{\cos^2 3x}$ b) $\int \frac{dx}{\sin^2 4x}$

$$\text{v)} \int \frac{dx}{\cos^2 25x} \quad \text{g)} \int \frac{dx}{\cos^2 4x} \quad \text{d)} \int \frac{dx}{\sin^2 3x}$$

Quyidagi trigonometrik formulalarni bilishimiz lozim:

$y' = (tgx)' = \frac{1}{\cos^2 x}$, $y' = (tg2x)' = \frac{2}{\cos^2 2x}$,

$y' = (ctgx)' = -\frac{1}{\sin^2 x}$, $y' = (ctg2x)' = -\frac{2}{\sin^2 2x}$.

$$\int \frac{dx}{\cos^2 x} = tgx + C; \quad \int \frac{dx}{\sin^2 x} = -ctgx + C.$$

a) $\int \frac{dx}{\cos^2 3x} = \frac{1}{3} tg3x + C$. b) $\int \frac{dx}{\sin^2 4x} = -\frac{1}{4} ctg4x + C$.

v) $\int \frac{dx}{\cos^2 25x} = \frac{1}{25} tg25x + C$. d) $\int \frac{dx}{\sin^2 3x} = -\frac{1}{3} ctg3x + C$.

$$\text{g)} \int \frac{dx}{\cos^2 4x} = \frac{1}{4} tg4x + C.$$

FOYDALANILGAN ADABIYOTLAR RO‘YXATI: (REFERENCES)

1. E.Xolmurodov, A.I.Yusupov. “Oliy matematika”-o‘quv qo‘llanma. “NOSHIR”-Toshkent-2013.

2. S.Alixonov. “Matematika o‘qitish metodikasi”-o‘quv qo‘llanma. Cho‘lpon nomidagi nashriyot matbba ijodiy uyi. Toshkent-2011.

3. A. A. Заитов. Элементы дифференциального исчисления. Учебное пособие. – Ташкент.