TEKISLIKDA SOMILIAN – BETTI FORMULASI

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ANNOTATSIYA

Bu ishda momentli elastikliklik nazariyasi tenglamalari sistemasi yechimini fazoda yechim va uning kuchlanishi soha chegarasining musbat o‘lchovli qismida berilganda sohaning ichiga topish masalasi qaraladi. Bunday masalaga Koshi masalasi deyiladi. Qaralayotgan masalaning yechimi mavjudligi kriteriyasi keltiriladi.

Kalit so‘zlar: Momentli elastikliklik nazariyasi, Karleman funksiyasi, Karleman matritsasi, Somilion-betti, Koshi masalasi.

KIRISH

Elastiklik elastiklik nazariyasi tenglamalari sistemasi tadqiqotning ob’ekti hisoblanib Koshi masalasi o‘rganiladi.

$x = (x_1, x_2),\ u = (y_1, y_2)$ nuqtalar R^2 tekislikdan olingan bo‘lsin va D elastik muhit R^2 da bo‘lakli – silliq ∂D chiziq bilan chegaralangan sohadan iborat bulsin.

Bir jinsli izotrop elastik muhitning asosiy tenglamasini ko‘rib chiqamiz, qaysiki vektor shaklida quyidagi ko‘rinishda yoziladi:

$$\mu\ \Delta\ u(x) + (\lambda + \mu)\ grad\ div\, u(x) = 0, \qquad (1)$$

bu yerda $u = (u_1;\, u_2)$ – ko‘chish vektori, λ, μ – ko‘rilayotgan elastik muxitning Lame doimiysi, Δ – Laplas operatori, bunda $\lambda, \mu > 0$.

(1) sistema ko‘rinishida quyidagicha yoziladi:

$$\left.\begin{aligned} \mu\ \Delta u_1(x) + (\lambda + \mu)\left[\frac{\partial^2 u_1(x)}{\partial x_1^2} + \frac{\partial^2 u_2(x)}{\partial x_1 \partial x_2}\right] = 0 \\ \mu\ \Delta u_2(x) + (\lambda + \mu)\left[\frac{\partial^2 u_2(x)}{\partial x_2^2} + \frac{\partial^2 u_1(x)}{\partial x_1 \partial x_2}\right] = 0 \end{aligned}\right\} \qquad (2)$$

(2) sistemani qisqa ifodalash uchun uni matrisa shaklida yozish qulay.

Shu maqsadda differenstial matrisali operatorni kiritamiz:

$$A\,(\partial_x) = \left\|A_{ij}(\partial_x)\right\|_{2x2}\ .$$

Bu yerda $A_{ij}(\partial_x) = \delta_{ij}\mu\,\Delta + (\lambda + \mu)\dfrac{\partial^2}{\partial x_i \partial x_j}$, bunda δ_{ij} – Kronekker simvoli.

U holda (1.2) sistemani quyidagicha tarzda matrisali shaklda yozish mumkin: $A\,(\partial_x)\,u(x) = 0.$ $u = (y_1, y_2)$ – nuqta D, elastik muhitning nuqtasi bo‘lsin, $n(u) = (n_1(u);\, n_2(u))$ esa y nuqtadagi normal birlik vektor. $n(u)$ yo‘nalish bo‘yicha y

nuqtadagi kuchlanishni hisoblash uchun quyidagi formuladan foydalanamiz: $T(\partial_y, n(u))\, u(u) \qquad (yoki\ T(\partial_y, n)\, u(u)\,)$

Bu yerda $T\left(\partial_y, n(u)\right) -$ matristali differenstial operator bo‘lib,

$$T\left(\partial_y, n(u)\right) = \left\|T_{ij}\left(\partial_y, n(u)\right)\right\|_{2\times 2} = \left\|\lambda n_i \frac{\partial}{\partial y_j} + \mu\, n_j \frac{\partial}{\partial y_i} + \mu\, \delta_{ij} \frac{\partial}{\partial n(u)}\right\|_{2\times 2},$$

Bunda $\frac{\partial}{\partial n(u)} = n_1 \frac{\partial}{\partial y_1} + n_2 \frac{\partial}{\partial y_2}$. $T\left(\partial_y, n(u)\right) -$ ni kuchlanish operatori deb ataymiz.

1-ta’rif. D da aniqlangan φ funksiyani D da regulyar deb ataymiz. Agar $\varphi \in C^1(\bar{D}) \cap C^2(D)\left(\varphi_{ij} \in C^1(\bar{D}) \cap C^2(D), i \times j = n \times m\right)$ va

$$\varphi\left(\varphi_{ij,} i \times j = n \times m\right)$$

funksiyaning Dekart koordinatalari bo‘yicha barcha ikkinchi tartibli hosilalari D sohada integrallanuvchi bo‘lsa.

Bundan keyin ∂D ni chekli D sohani chegaralovchi yopiq silliq egri chiziq deb hisoblaymiz.

Endi ushbu ishda o‘rganiladigan elastiklik nazariyasi sistemasi uchun Koshi masalasining qo‘yilishini bayon etish mumkin. $u(y) = (u_1(y); u_2(y)) -$ (2) sistemaning D sohadagi regulyar yechimi bo‘lsin, ya’ni

$$A\left(\partial_y\right)u(y) = 0 \tag{3}$$

$$\left.\begin{aligned} u(y) &= f(y), \qquad y \in S \\ T\left(\partial_y, n\right)u(y) &= g(y), \quad y \in S \end{aligned}\right\} \tag{4}$$

Bunda $S - \partial D$ ning qismi, $f(y) = (f_1(y); f_2(y))$ va $g(y) = (g_1(y); g_2(y))$ lar esa S da berilgan uzluksiz vektor – funksiyalar. $n = (n_1; n_2) -$ y nuqtada S ga nisbatan ortonormal vektor.

Berilgan $f(y)$ va $g(y)$ lardan kelib chiqib D da $u(u)$ ni tiklash talab qilinadi.

(1) sistema elliptik. (3) sistemaning elliptikligi shundan iboratki,

${\xi_1}^2 + {\xi_2}^2 \neq 0$ tenglikni qanoatlantiruvchi ixtiyoriy haqiqiy ξ_1 va ξ_2 lar uchun $detA(\xi)$ determinant noldan farqli bo‘lsin, bu erda $\xi = (\xi_1; \xi_2)$, lekin

$$detA(\xi) = \mu(\lambda + 2\mu)\left({\xi_1}^2 + {\xi_2}^2\right)^2$$

Shuning uchun, $\lambda, \mu > 0$ ni hisobga olib (3) elliptik sistema bo‘lishini tasdiqash mumkin.

Misol. $U(y) = \left(u_1(y); u_2(y)\right)$ vektor – funksiya $y_2 > 0$ yarim tekislikda elastiklik nazariyasi sistemasining regulyar yechimi bo‘lsin, ya’ni $A\left(\partial_y\right)U(y) = 0$

va Koshi shartini qanoatlantirsin: $u(y_1, 0) = \left(0, \frac{cos\,(\sigma\, u_1)}{\sigma^2}\right)$,

$T\left(\partial_y, n(y)\right) u(y_1, 0) = \left(0, \frac{2(\lambda+\mu)\cos(\sigma u_1)}{\sigma}\right)$ $y_2 = 0$ ning chegarasida $|u(y)| = \frac{|\cos(\sigma u_1)|}{\sigma^2}$, $\left|T\left(\partial_y, n(y)\right) u(y)\right| = \frac{2(\lambda+\mu)|\cos(\sigma u_1)|}{\sigma}$.

bo‘lishi tushunarli. Shunday qilib, yetarlicha katta σ da, $u(y)$ yechim uchun boshlang‘ich shartlar kichik bo‘ladi,

$$u(u) = \left(\frac{\sin(\sigma u_1)\, sh\,(\sigma u_2)}{\sigma^2}, \qquad \frac{\cos(\sigma u_1)\, ch\,(\sigma u_2)}{\sigma^2}\right),$$

Yechimning o‘zi esa $u_1 \neq 0$, $u_2 \neq 0$, $u_2 > 0$ larda keragicha katta qilingan bo‘lishi mumkin, yani turg‘unlik o‘rinli emas.

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