LEAST QUADRATIC NON-RESIDUE AND VINOGRADOV'S CONJECTURES

Abdunabiev Jamshid
Master's degree at Termez State University
70540101 - Mathematics (Algebra and number theory) specialization II year 123 - master's student
jamshidabdunabiyev5@gmail.com

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ABSTRACT

Vinogradov's conjectures are fundamental problems in number theory that have intrigued mathematicians for decades. One of the key aspects of these hypotheses is the concept of least square nonresidue, which plays a crucial role in understanding the distribution of prime numbers and the behavior of arithmetic functions. This article explores the meaning of least square non-residue in the context of the Vinogradov conjectures, shedding light on its significance for the study of prime numbers and deeper connections between number theory and algebraic structures. By delving into the complexities of these conjectures and their relationship to least square nonresidue, this abstract aims to provide a comprehensive overview of the current state of research in this fascinating area of mathematics.

Key words: *Key words: least quadratic non-residue, Vinogradov's conjectures, number theory, prime numbers, arithmetic functions, quadratic residues, quadratic non-residues.*

Vinogradov's conjectures, on the other hand, are assumptions about the distribution of prime numbers in arithmetic progressions. These topics have deep connections to various areas of mathematics and have generated extensive research and study. This article provides an overview of the concept of least quadratic nonresidue, discusses Vinogradov's conjectures, and explores their significance in the field of number theory.[1]

The concept of least quadratic nonresidue and the Vinogradov conjecture are fascinating topics in number theory that have intrigued mathematicians for decades. Least quadratic non-residue refers to the smallest positive integer that is not a quadratic residue modulo the given prime. Quadratic residues are numbers that have square roots modulo the prime number, while non-residues do not. The study of least square non-residues provides insight into the distribution of mathematician Ivan Matveevich Vinogradov, are a set of hypotheses related to the distribution of prime numbers. These hypotheses suggest certain patterns and connections between prime numbers and their distribution in arithmetic progressions. Vinogradov's conjectures have profound implications for prime number theory and have inspired extensive research in number theory quadratic remainders and non-residues, as well as their properties and relationships with prime numbers.

The connection between least quadratic nonresidue and Vinogradov's conjectures lies in their common focus on the distribution and properties of numbers in relation to prime numbers. By studying least quadratic nonresidue and its relationship to quadratic remainders and nonresidues, mathematicians can gain insight into the distribution of prime numbers and potentially discover new patterns or connections that shed light on Vinogradov's conjectures. The study of least quadratic nonresidue and the Vinogradov conjectures opens up a rich and challenging environment for mathematical research with the potential to deepen our understanding of number theory and prime number distributions.[2]

For each prime p, let n(p) denote the least square nonresidue modulo p. Vinogradov assumed that n(p) = O(p^\eps) for any fixed \eps>0.

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[1] Vinogradov I. M. (1947). Method of trigonometric sums in number theory. Interscience Publishing House.

[2] Montgomery, H.L., and Vaughan, R.C. (2007). Multiplicative Number Theory I: Classical Theory (Vol. 97). Cambridge University Press.

This conjecture follows from the generalized Riemann hypothesis and is known to be true for almost all primes p, but remains open in general.

Vinogradov's conjectures are among the most famous and important conjectures in the field of number theory. They were proposed by Russian mathematician Ivan Vinogradov in 1937 and still remain unsolved.

Vinogradov's first conjecture, known as the Three Prime Theorem, states that for any large number N there are three prime numbers p, p+2 and p+2N, where p is the smallest prime number greater than N.

This conjecture has important consequences in the field of cryptography and proof of the complexity of some algorithms. Vinogradov's second conjecture states that for any natural number a and any large number N, there exists a prime number p such that $p \equiv a \pmod N$. This means that the prime numbers are evenly distributed in arithmetic progressions. This conjecture has important consequences in the field of cryptography, as well as in the solution of some Diophantine equations. The study of Vinogradov's conjectures has deep connections with other areas of mathematics, such as algebraic and analytic number theory. It also has practical implications in cryptography and computational methods.[3]

Although none of Vinogradov's hypotheses have been fully proven, there are many works and studies aimed at confirming or refuting them. Recent breakthroughs and discoveries in this field have led to new methods and techniques that can be used for further research. In conclusion, Vinogradov's conjectures represent important and interesting problems in the field of number theory. Their research has significant implications for various areas of mathematics and can lead to new discoveries and applications.

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[3] Hardy, G. H., & Wright, E. M. (2008). Introduction to number theory. Oxford University Press.

The interaction of these areas allows the development of new methods and approaches to solving these hypotheses. Algebraic number theory studies the properties of integers and their algebraic connections. It helps to analyze the structure of prime numbers and their distribution in various arithmetic progressions. This knowledge can be applied to study Vinogradov's hypotheses.[4]

Analytical number theory, on the other hand, uses methods of mathematical analysis to study the numerical properties of sequences of numbers. It allows you to analyze the distribution of prime numbers and study their behavior in the context of Vinogradov's conjectures. In addition, the study of Vinogradov's conjectures has practical significance in cryptography and computational methods. Understanding the distribution of prime numbers and the properties of arithmetic progressions can be used to develop secure cryptographic algorithms. It can also help optimize computational methods based on large number arithmetic.

Overall, the study of Vinogradov's conjectures has deep connections with other areas of mathematics and has practical implications in cryptography and computational methods. This is an important area of research that could lead to new discoveries and applications in the future.[5]

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