KICHIK O'LCHAMLI LEYBNITS ALGEBRALARINING KVAZI-DIFFERENSIYALASHLARI VA ULARNING XOSSALARI

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ANNOTATSIYA

Maqolada kichik o'lchamli Leybnits algebralarining kvazi-differensiallashlari va ularning xossalari haqida olingan natijalar keltiriladi.

Kalit so'zlar: *Leybnits algebralari, differensiallash, kvazi-differensiallash, umumlashgan differensiallash, sentroid va kvazi-sentroidlar.*

ABSTRACT

The article presents the results obtained about quasi-differentiations of small-dimension Leibniz algebras and their properties.

Key words: *Leibniz algebras, differentiation, quasi-differentiation, generalized differentiation, centroids and quasi-centroids.*

АННОТАЦИЯ

В статье представлены полученные результаты о квазидифференцировки алгебр Лейбница малой размерности и их свойствах.

Ключевые слова: *Алгебры Лейбница, дифференцирование, квазидифференцировки, обобщенное дифференцирование, центроиды и квазицентроиды.*

KIRISH

Hozirgi kunda Li algebralarning umumlashmasi hisoblangan Leybnits algebralari sinfi jadal suratda o‘rganilmoqda. Ta‘kidlash joizki, Leybnits ayniyatini qanoatlantiruvchi algebralar birinchi bo‘lib 1965-yilda A.Bloxning ishida D-algebralar nomi bilan kiritilgan edi. Lekin, D-algebralarni o‘rganishga unchalik e‘tibor berilmagan bo‘lib, faqatgina J.L. Lode va T.Pirashvilining ishlaridan keyingina Leybnits algebralari jadal suratda o‘rganila boshlandi va hozirgi kunga kelib bu algebralarga bag‘ishlangan bir qator maqolalar chop qilindi Leybnits algebralari o‘tgan asrning 90-yillarida fransuz matematigi J.L. Lode tomonidan ushbu

$$[x,[y,z]] = [[x,y],z] - [[x,z],y]$$

Leybnits ayniyati bilan xarakterlanadigan algebra sifatida fanga kiritilgan. 1998-yildan boshlab Leybnits algebrasining strukturaviy nazariyasini Sh.A. Ayupov va B.A. Omirovlar o‘rgana boshladi. Algebraning o‘lchami qancha kattalashgan sari, uni tavsiflash shuncha murakkab bo‘ladi. Nilpotent Leybnits algebralari bilan Ayupov Sh.A., Omirov B.A., Raximov I.S., Rixsiboev I.M., Xudoyberdiyev A.X. va boshqalar shug‘ullangan. Katta o‘lchamdagi nilpotent Li algebralarini ham o‘rganish murakkab bo‘lgani uchun, nilpotent algebralar bir necha sinflarga bo‘linadi. Masalan, nol filiform, filiform, kvazi filiform va boshqa sinflar.

So‘nggi yillarda noassotsiativ algebralarning differensiallashlari va differensiallashlarning umumlashmalari hisoblangan qator operatorlar keng o‘rganilmoqda. Xususan, kvazi-differensiallashlar tushunchalari operator algebralaridan tashqari Li va Leybnits algebralari uchun ham o‘rganildi. Ushbu maqolada kichik o‘lchamli Leybnits algebralarining kvazi-differensiallashlari tushunchasi o‘rganiladi. Kichik o‘lchamli Leybnits algebralarining kvazi-differensiallashlari va ularning xossalari aniqlanadi.

Ta’rif 1. F maydonda berilgan $(L,[-,-])$ algebraning ixtiyoriy x,y,z elementlari uchun quyidagi Leybnits ayniyati o‘rinli bo‘lsa:

$$[x,[y,z]] = [[x,y],z] - [[x,z],y],$$

u holda $(L,[-,-])$ algebra Leybnits algebrasi deb ataladi.

Ta’rif 2. Aytaylik, $d: L \to L$ chiziqli akslantirish bo‘lsin. Agar $(L, [-,-])$ Leybnits algebrasining ixtiyoriy elementlari uchun quyidagi tenglik bajarilsa,

$$d([x, y]) = [d(x), y] + [x, d(y)],$$

u holda d chiziqli akslantirish L Leybnits algebrasining differensiallashi deyiladi.

Barcha differensiallashlar to‘plamini $Der(L)$ kabi belgilaymiz.

Ta’rif 3. Agar $D \in End(L)$ akslantirish uchun, $\exists D', D'' \in End(L)$ akslantirishlar topilib, $\forall x, y \in L$ elementlar uchun quyidagi ayniyat bajarilsa,

$$[D(x), y] + [x, D'(y)] = D''([x, y])$$

u holda D akslantirishga L Leybnits algebrasining **umumlashgan differensillashi** deyiladi.

Ta’rif 4. Agar $D \in End(L)$ akslantirish uchun, $\exists D' \in End(L)$ akslantirish topilib, $\forall x, y \in L$ elementlar uchun quyidagi ayniyat bajarilsa,

$$[D(x), y] + [x, D(y)] = D'([x, y])$$

u holda D akslantirishga L Leybnits algebrasining **kvazi-differensillashi** deyiladi.

L Leybnits algebrasining barcha umumlashgan va kvazi differensiallashlari to‘plami mos ravishda $GDer(L)$ va $QDer(L)$ kabi belgilanadi. Ta‘kidlash joizki, ixtiyoriy differensiallash kvazi differensiallash bo‘ladi. Biroq, kvazi-differensiallashlar oddiy differensiallash bo‘lmasligi mumkin.

Endi algebraning sentroidi, kvazi-sentoidi va sentral differensiallashlari tushunchalarini aniqlaymiz.

Ta’rif 5. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)] = D([x, y])$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **sentroidi** deyiladi. Barcha sentroidlar to‘plamini $C(L)$ bilan belgilanadi.

Ta’rif 6. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)]$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **kvazi-sentroidi** deyiladi. Barcha kvazi-sentroidlar to‘plamini $QC(L)$ bilan belgilanadi.

Ta’rif 7. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)] = D([x, y]) = 0$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **sentral differensiallashi** deyiladi. Barcha sentral differensiallashlar to‘plamini $ZDer(L)$ bilan belgilanadi. Ma‘lumki,

$$ZDer(L) \subseteq Der(L) \subseteq QDer(L) \subseteq GDer(L) \subseteq End(L)$$

munosabat o‘rinli bo‘ladi. Shuningdek,

$$C(L) \subseteq QC(L) \subseteq QDer(L)$$

munosabat ham o‘rinli bo‘ladi.

Ikki o‘lchamli Leybnits algebralarining kvazi-differensiallashlari:

Ma’lumki, har qanday ikki-o‘lchamli Leybnits algebrasi quyidagi izomorf bo‘lmagan Leibnits algebralaridan biriga izomorf:

$L_1: [e_1, e_1] = e_2$

$L_2: [e_1, e_2] = -[e_2, e_1] = e_2$

$L_3: [e_1, e_2] = [e_2, e_2] = e_1$

NATIJALAR:

Biz shu uch xil 2-o‘lchamli algebralarning barcha differensiallashlari, kvazi-differensiallashlari, sentroidi, kvazi-sentroidi va umumlashgan differensiallashlarini umumiy ko‘rinishlarini aniqlaymiz:

Teorema 1. $L_1: [e_1, e_1] = e_2$ algebraning differensiallashlari quyidagicha ko‘rinishga ega:

$L_1: [e_1, e_1] = e_2$				
$Der(L_1)$	$QDer(L_1)$	$GDer(L_1)$	$C(L_1)$	$QC(L_1)$
$\begin{pmatrix} d_{11} & d_{12} \\ 0 & 2d_{11} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{11} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix}$

Teorema 2. $L_2: [e_1, e_2] = -[e_2, e_1] = e_2$ algebraning differensiallashlari quyidagicha ko‘rinishga ega:

$L_2: [e_1, e_2] = -[e_2, e_1] = e_2$

$Der(L_2)$	$QDer(L_2)$	$GDer(L_2)$	$C(L_2)$	$QC(L_2)$
$\begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$

Teorema 3. $L_3: [e_1, e_2] = [e_2, e_2] = e_1$ algebraning differensiallashlari quyidagicha ko‘rinishga ega:

$L_3: [e_1, e_2] = [e_2, e_2] = e_1$				
$Der(L_3)$	$QDer(L_3)$	$GDer(L_3)$	$C(L_3)$	$QC(L_3)$
$\begin{pmatrix} d_{11} & 0 \\ d_{11} & 0 \end{pmatrix}$	$\begin{pmatrix} d_{21} + d_{22} & 0 \\ d_{21} & d_{22} \end{pmatrix}$	$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$

Uch o‘lchamli nilpotent Leybnits algebralarining kvazi-differensiallashlari:

Bizga quyidagi uch o‘lchamli nilpotent Leybnits algebralari berilgan bo‘lsin:

λ_1: $abelian$;

λ_2: $[e_1, e_1] = e_2$;

λ_3: $[e_2, e_3] = e_1, \ [e_3, e_2] = -e_1$;

λ_4: $[e_2, e_1] = e_3, \ [e_1, e_2] = \alpha e_3, \ \alpha \neq \alpha^{-1} (\alpha \in C)$;

λ_5: $[e_1, e_1] = e_3, \ [e_2, e_1] = e_3, \ [e_1, e_2] = -e_3$

λ_6: $[e_1, e_1] = e_2, \ [e_2, e_1] = e_3$.

Bu algebralar uchun kvazi-differensiallashlar to‘plamini topamiz.

Tasdiq 1. $\lambda_1: abelian$ algebraning barcha differensiallashlari fazosi matritsasining umumiy ko‘rinishi quyidagicha bo‘ladi:

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

Teorema 4. λ_2: $[e_1, e_1] = e_2$ algebralar uchun kvazi-differensiallashlar fazosi matritsasining umumiy ko‘rinishi quyidagicha bo‘ladi:

$$QDer(\lambda_2) = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \beta_2 & \beta_3 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$$

λ_2 algebraning qolgan differensiallashlarini isbotsiz quyidagi jadvalda keltiramiz:

$\lambda_2: [e_1, e_1] = e_2$			
$Der(\lambda_2)$	$GDer(\lambda_2)$	$C(\lambda_2)$	$QC(\lambda_2)$
$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 2\alpha_1 & 0 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \beta_2 & \beta_3 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \alpha_1 & 0 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \beta_2 & \beta_3 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$

Teorema 5. λ_3: $[e_2, e_3] = e_1$, $[e_3, e_2] = -e_1$ algebralar uchun kvazi-differensiallashlar fazosining matritsasi quyidagi ko‘rinishda bo‘ladi:

$$QDer(\lambda_3) = \begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

$\lambda_3: [e_2, e_3] = e_1, [e_3, e_2] = -e_1$			
$Der(\lambda_3)$	$GDer(\lambda_3)$	$C(\lambda_3)$	$QC(\lambda_3)$
$\begin{pmatrix} \beta_2+\gamma_3 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \alpha_1 & 0 \\ \gamma_1 & 0 & \alpha_1 \end{pmatrix}$	$\begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \beta_2 & 0 \\ \gamma_1 & 0 & \gamma_3 \end{pmatrix}$

Teorema 6. λ_4: $[e_2, e_1] = e_3$, $[e_1, e_2] = \alpha e_3$, $\alpha \neq \alpha^{-1}, (\alpha \in C)$ algebralar uchun kvazi-differensiallashlar fazosining matritsasi quyidagi ko‘rinishda bo‘ladi:

$$QDer(\lambda_4) = \begin{pmatrix} \alpha_1 & 0 & \alpha_3 \\ 0 & \beta_2 & \beta_3 \\ 0 & 0 & \gamma_3 \end{pmatrix}$$

λ_4: $[e_2,e_1]=e_3,\ [e_1,e_2]=\alpha e_3,\ \alpha\neq\alpha^{-1},(\alpha\in C)$			
$Der(\lambda_4)$	$GDer(\lambda_4)$	$C(\lambda_4)$	$QC(\lambda_4)$
$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \beta_2 & \beta_3\\ 0 & 0 & \alpha_1+\beta_2\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & \alpha_1\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \beta_2 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$

Teorema 7. λ_5: $[e_1,e_1]=e_3,\ [e_2,e_1]=e_3,\ [e_1,e_2]=-e_3$ algebralar uchun kvazi-differensiallashlar fazosining matritsasi quyidagicha bo‘ladi:

$$QDer(\lambda_5)=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$$

λ_5: $[e_1,e_1]=e_3,[e_2,e_1]=e_3,[e_1,e_2]=-e_3$			
$Der(\lambda_5)$	$GDer(\lambda_5)$	$C(\lambda_5)$	$QC(\lambda_5)$
$\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & 2\alpha_1\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ 0 & \beta_2 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & \alpha_1\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$

Teorema 8. λ_6: $[e_1,e_1]=e_2,\ [e_2,e_1]=e_3$ algebralar uchun kvazi-differensiallashlar fazosining matritsasi quyidagicha bo‘ladi:

$$QDer(\lambda_6)=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ 0 & \beta_2 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$$

λ_6: $[e_1,e_1]=e_2,[e_2,e_1]=e_3$			
$Der(\lambda_6)$	$GDer(\lambda_6)$	$C(\lambda_6)$	$QC(\lambda_6)$
$\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ 0 & 2\alpha_1 & \alpha_2\\ 0 & 0 & 3\alpha_1\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_1 & 0\\ 0 & 0 & \alpha_1\end{pmatrix}$	$\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_1 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}$

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