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AYRIM IRRATSIONAL KO‘RINISHDAGI FUNKSIYALARNI TRIGONOMETRIK ALMASHTIRISHLAR YORDAMIDA INTEGRALLASH

Ibrohimov Javohir Bahrom o‘g‘li
O‘zbekiston Milliy universiteti Jizzax filiali assistenti

Po‘latov Baxtiyor Sobirovich
O‘zbekiston Milliy universiteti Jizzax filiali katta o‘qituvchisi

Ataqulova Fotima Qodirboy qizi
O‘zbekiston Milliy universiteti Jizzax filiali talabasi

Soibova Ruxshona Xurshid qizi
O‘zbekiston Milliy universiteti Jizzax filiali talabasi

Annotatsiya: *Ayrim murakkab ko‘rinishdagi ifodalarni integrallash davomida integral ostidagi irratsional funksiyaning ko‘rinishiga alohida e’tibor beramiz. Quyidagi:* $\int R(x,\sqrt{a^2-x^2})dx$, $\int R(x,\sqrt{a^2+x^2})dx$, $\int R(x,\sqrt{x^2-a^2})dx$ *ko‘rinishdagi integrallar, mos ravishda* $x = a\sin t, x = a\tan t, x = a\sec t, a \in R, a \neq 0$, *almashtirishlar natijasida ratsionallashtirilib hisoblanadi*

Kalit so‘zlar: *Integral, irratsional funksiya,trigonometrik almashtirishlar.*

$\int R(x,\sqrt{a^2-x^2})dx$, $\int R(x,\sqrt{a^2+x^2})dx$, $\int R(x,\sqrt{x^2-a^2})dx$ *ko‘rinishdagi ifodalarni integirallashni misollarda batafsil ko‘rib chiqamiz.*

Misollar:

1-misol. $\int(\sqrt{a^2-b^2x^2})dx=$ $\left[t=\arcsin\left(\frac{x}{\frac{a}{b}}\right);\ x=\frac{a}{b}sint;\ dx=\frac{a}{b}\cos tdt\right]$

$$\int\sqrt{a^2-b^2(\frac{a}{b}sint)^2}\,\frac{a}{b}\cos tdt=\frac{a}{b}\int\sqrt{a^2-b^2(\frac{a}{b}sint)^2}\,\frac{a}{b}\cos tdt=$$

$$\frac{a}{b}\int\sqrt{a^2-b^2\frac{a^2}{b^2}sin^2t}\cos tdt=$$

$\frac{a}{b}\int\sqrt{a^2(1-sin^2t)}\cos tdt$=$\frac{a}{b}\int\sqrt{a^2\cos t^2 t}\cos tdt$=$\frac{a}{b}\int a*\cos t*\cos tdt$=

$\frac{a^2}{b}\int\frac{1+\cos 2t}{2}dt$=$\frac{a^2}{2b}\int(1+\cos 2t)dt=\frac{a^2}{2b}\left(t+\frac{\sin 2t}{2}\right)$=$\frac{a^2}{2b}\left(\arcsin\left(\frac{x}{\frac{a}{b}}\right)+\frac{sin\left(\arcsin\left(\frac{x}{\frac{a}{b}}\right)\right)}{2}\right)$=$\frac{a^2arcsin\frac{bx}{a}+\frac{bx\sqrt{a^2-b^2x^2}}{x}}{2b}$+C

2-misol. $\int\frac{\sqrt{4+x^2}}{x^2}$dx= $\left[x=2tgt;\ t=arctg\frac{x}{2};dx=2*\frac{1}{cos^2t}dt\right]$=

$\int\frac{\sqrt{4+4tg^2t}}{8tg^3t}*\frac{2}{cos^2t}$dt=$\int\frac{\sqrt{1+tg^2t}}{2\frac{\sin^3t}{cost}}$

dt=$\int\frac{\frac{1}{cost}}{2\frac{\sin^3t}{cost}}$dt=$\int\frac{1}{2\sin^3t}$dt=$\frac{1}{2}\int\frac{sin^2t+cos^2t}{sin^3t}$dt=$\frac{1}{2}\int\left(\frac{1}{sint}+\frac{cos^2t}{sin^3t}\right)$dt=$\frac{1}{2}\int\frac{1}{sint}dt$+$\frac{1}{2}\int\frac{cos^2t}{sin^3t}dt=\frac{1}{2}\int\frac{1}{sint}dt+\frac{1}{2}\int cost*\frac{cost}{sin^3t}$dt=

=$\begin{bmatrix}\mathrm{u}=\mathrm{cost} & \mathrm{v}=-\frac{1}{2sin^2t}\\ du=-sintdt & dv=\frac{cost}{sin^3t}dt\end{bmatrix}$=$\frac{1}{2}\int\frac{1}{sint}\mathrm{dt}+\frac{1}{2}\cos t$*$(-\frac{1}{2sin^2t})$-

$\frac{1}{2}\int-\frac{1}{2sin^2t}(-\sin t)$dt=-

$\frac{\cos t}{4sin^2t}$+$\frac{1}{4}\int\frac{1}{sint}$dt=$-\frac{1}{4}\frac{\cos t}{sin^2t}$+$\frac{1}{4}\int\frac{\sin t}{sin^2t}$dt=$-\frac{1}{4}\frac{\cos t}{sin^2t}$+$\frac{1}{4}\int\frac{\sin t}{1-cos^2t}$dt= $[u=cost]$

=-$\frac{1}{4}\frac{\cos t}{sin^2t}$+$\frac{1}{4}\int\frac{1}{u^2-1}$du==-$\frac{1}{4}\frac{\cos t}{sin^2t}$+$\frac{1}{4}\int\frac{1}{2}(\frac{1}{u-1}-\frac{1}{1+u})$du =-$\frac{1}{4}\frac{\cos t}{sin^2t}+\frac{1}{8}$(ln$|u-1|$-ln$|1+u|$)= -$\frac{1}{4}\frac{\cos t}{sin^2t}$+$\frac{1}{8}\ln\left|\frac{cost-1}{cost+1}\right|$=$-\frac{1}{4}\frac{\cos(arctg\frac{x}{2})}{sin^2(arctg\frac{x}{2})}$-$\frac{1}{8}\ln\left|tg^2(\frac{arctg\frac{x}{2}}{2})\right|$=

$$\begin{bmatrix}\cos(arctg\frac{x}{2})=\frac{1}{\sqrt{1+\frac{x^2}{4}}}=\frac{2}{\sqrt{4+x^2}}\\ sin^2(arctg\frac{x}{2})=\frac{x^2}{4+x^2}\end{bmatrix}$$

$=-\frac{1}{4}(\frac{\frac{2}{\sqrt{4+x^2}}}{\frac{x^2}{4+x^2}}+\frac{1}{2}\ln\left|\frac{8+x^2+4\sqrt{4+x^2}}{x^2}\right|)= =-\frac{1}{4}\left(\frac{2\sqrt{4+x^2}}{x^2}+\frac{1}{2}\ln\left|\frac{8+x^2+4\sqrt{4+x^2}}{x^2}\right|\right)+C$

3-misol. $\int\frac{\sqrt{a^2+b^2x^2}}{x^2}$dx=

$$\left[\begin{matrix} x=\frac{a}{b}tgt & t=arctg\frac{x}{\frac{a}{b}} \\ dx=\frac{a}{b}\frac{1}{cos^2t}dt & \end{matrix}\right]$$

$=\int\frac{\sqrt{a^2+b^2\frac{a^2}{b^2}tg^2t}}{\frac{a^2}{b^2}tg^2t}*\frac{a}{b}\frac{1}{cos^2t}dt=\int\frac{a\sqrt{1+tg^2t}}{\frac{a^2}{b^2}*\frac{sin^2t}{cos^2t}}*\frac{a}{b}\frac{1}{cos^2t}$dt=

$b\int\frac{1}{\sqrt{cos^2t}}*\frac{1}{sin^2t}$dt=b$\int\frac{1}{sin^2t(cost)}dt=$

$b\int\frac{cost}{sin^2tcos^2t}$dt=b$\int\frac{cost}{sin^2tcos^2t}dt=b\int\frac{cost}{(1-sin^2t)sin^2t}$dt=

$$[u=sint]$$

=b $\int\frac{1}{(1-u^2)u^2}$du=

$$\left[\begin{matrix} \frac{1}{(1-u^2)u^2}=\frac{A}{(1-u)}+\frac{B}{(1+u)}+\frac{C}{u^2}+\frac{D}{u} \\ 1=u^2(1+u)A+(1-u)u^2B+(1-u^2)C+(1-u^2)uD \\ A=\frac{1}{2};D=0,B=\frac{1}{2};C=1 \end{matrix}\right]$$

= b $\int(\frac{1}{2(1-u)}+\frac{1}{2(1+u)}+\frac{1}{u^2})du$=b(-$\frac{1}{2}\ln|1-u|+\frac{1}{2}\ln|1+u|-\frac{1}{u}$)=

b(-$\frac{1}{2}\ln\left|\frac{1-u}{1+u}\right|-\frac{1}{u}$)=b(-$\frac{1}{2}\ln\left|\frac{1-sint}{1+sint}\right|-\frac{1}{sint}$)=b(-$\frac{1}{2}\ln\left|\frac{1-\sin(arctg\frac{xb}{2})}{1+\sin(arctg\frac{xb}{2})}\right|-\frac{1}{\sin(arctg\frac{xb}{2})}$) $=$

$$\left[\sin\left(\operatorname{arctg}\left(\frac{xb}{a}\right)\right)=\frac{bx}{\sqrt{a^2+b^2x^2}}\right]$$

= b(-$\frac{1}{2}\ln\left|\frac{1-\frac{bx}{\sqrt{a^2+b^2x^2}}}{1+\frac{bx}{\sqrt{a^2+b^2x^2}}}\right|-\frac{\sqrt{a^2+b^2x^2}}{bx}$) $+C$ C$\in R$;

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