TRIGONOMETRIK FUNKSIYALARNING BA'ZI XOSSALARI

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Annotatsiya: Ushbu maqolada trigonometrik ba'zi funksiyalarni birini ikkinchisi orqali ifodalsh, trigonometrik formulalarning ba'zi isbotlari va trigonometrik funksiyalarga oid namunaviy masalalar keltirilib o'tilgan.

Kalit so'zlar: trigonometrik funksiya, ayniy shakl almashtirishlar, keltirish formulalari, ikkilangan burchak tushunchasi.

Maktab matematika kursida trigonometrik ifodalarning juda ko'p ayniy almashtirishlari mavjud bo'lib quyidagilar o'rganiladi.

- Trigonometrik funksiyalarni birini ikkinchisi orqali ifodalovchi ayniy almashtirishlar.
- Trigonometrik ifodalarni soddalashtirishdagi ayniy almashtirishlar.
- Trigonometrik ayniyatlarni hisoblashdagi ayniy almashtirishlar.
- Trigonometrik tenglamalarning yechimini topishdagi ayniy almashtirishlar.

Quyidagi formulalarga e'tibor qarataylik.

Keltirish formulalari:

1. $\sin\left(\frac{\pi}{2} \pm \alpha\right) = cos\alpha$

2. $\sin(\pi \pm \alpha) = \mp cos\alpha$

3. $\cos\left(\frac{\pi}{2} \pm \alpha\right) = \mp sin\alpha$

4. $\cos(\pi \pm \alpha) = -cos\alpha$

5. $\sin\left(\frac{3\pi}{2} \pm \alpha\right) = -cos\alpha$

6. $\cos\left(\frac{3\pi}{2} \pm \alpha\right) = \mp sin\alpha$

7. $\text{tg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp ctg\alpha$

8. $\text{ctg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp tg\alpha$

Asosiy trigonometrik ayniyatlar:

1. $cos^2\alpha + sin^2\alpha = 1$

   Isboti: Ushbu asosiy ayniyatni isbotlashda Pifagor teoremasidan foydalanamiz.

2. $tg\,\alpha = \frac{sin\alpha}{cos\alpha}$

3. $ctg\,\alpha = \frac{cos\alpha}{sin\alpha}$

4. $ctg\,\alpha \cdot tg\,\alpha = 1$

5. $\frac{1}{cos^2\alpha} = 1 + tg^2\alpha$

6. $\frac{1}{sin^2\alpha} = 1 + ctg^2\alpha$

Ikki burchak yig'indi va ayirmasining trigonometrik funksiyasi:

1. $\sin(\alpha \pm \beta) = sin\alpha cos\beta \pm cos\alpha sin\beta$

2. $\cos(\alpha \pm \beta) = cos\alpha cos\beta \mp sin\alpha sin\beta$

3. $\text{tg}(\alpha + \beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{sin\alpha cos\beta - cos\alpha sin\beta}{cos\alpha cos\beta - sin\alpha sin\beta} = \frac{tg\alpha cos\beta + 1 sin\beta}{cos\beta - tg\,\alpha sin\beta} = \frac{tg\alpha + tg\beta}{1 - tg\alpha tg\beta}$

4. $\text{tg}(\alpha - \beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha tg\beta}$

5. $\text{ctg}(\alpha + \beta) = \frac{1}{\text{tg}(\alpha-\beta)} = \frac{1 - tg\alpha tg\beta}{tg\alpha + tg\beta} = \frac{'1 - \frac{1}{ctg\alpha}\cdot\frac{1}{ctg\beta}}{\frac{1}{ctg\alpha} + \frac{1}{ctg\beta}} = \frac{ctg\alpha ctg\beta - 1}{ctg\alpha + ctg\beta}$

6. $\text{ctg}(\alpha - \beta) = \frac{ctg\alpha ctg\beta + 1}{ctg\alpha - ctg\beta}$

Ikkalangan burchak trigonometrik funksiyalari

1. $sin2\alpha = 2sin\alpha cos\alpha$

Isbot: $\sin 2\alpha = \sin(\alpha + \alpha) = sin\alpha cos\alpha + cos\alpha sin\alpha = 2sin\alpha cos\alpha$

Formula isbotlandi.

2. $cos2\alpha = cos^2\alpha - sin^2\alpha$

3. $tg2\alpha = \frac{2tg\,\alpha}{1 - tg^2\alpha}$

4. $ctg2\alpha = \frac{ctg^2\alpha - 1}{2ctg\alpha}$

5. $cos^2\alpha = \frac{1 + cos2\alpha}{2}$

6. $sin^2\alpha = \frac{1 - cos2\alpha}{2}$

Trigonometrik ko'paytmani yig'indiga keltirish:

1. $sin\alpha cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

   Isbot: $sin\alpha cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

   $+\begin{cases}\sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta \\ \sin(\alpha - \beta) = sin\alpha cos\beta - cos\alpha sin\beta\end{cases}$

   $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2sin\alpha cos\beta$

   $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2sin\alpha cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

2. $cos\alpha cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

3. $sin\alpha sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

Yig'indi va ayirmani ko'paytmaga keltirish:

1. $sin\alpha \pm sin\beta = 2 \cdot sin\frac{\alpha \pm \beta}{2} cos\frac{\alpha \pm \beta}{2}$

2. $cos\alpha + cos\beta = 2 \cdot cos\frac{\alpha + \beta}{2} cos\frac{\alpha - \beta}{2}$

3. $cos\alpha - cos\beta = 2 \cdot sin\frac{\alpha + \beta}{2} sin\frac{\alpha - \beta}{2}$

Isbot: $sin\frac{\alpha + \beta}{2} cos\frac{\alpha - \beta}{2} = \frac{1}{2}\left[\sin\left(\frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2}\right) + \sin\left(\frac{\alpha + \beta}{2} - \frac{\alpha - \beta}{2}\right)\right] = \frac{1}{2}(sin\alpha +$

$sin\beta) \Rightarrow [sin\alpha + sin\beta = 2 \cdot sin\frac{\alpha \pm \beta}{2} cos\frac{\alpha \pm \beta}{2}] \Rightarrow sin\alpha + sin\beta$

Trigonometrik funksiya trigonometrik yarim burchak bo'yicha formula

1. $sin\alpha = \frac{2tg\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$

Isbot: $sin\alpha = \frac{2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2}} = \frac{\frac{2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}{cos^2\frac{\alpha}{2}}}{\frac{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2}}{cos^2\frac{\alpha}{2}}} = \frac{2tg\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$

2. $cos\alpha = \frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$

3. $tg\alpha = \frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$

Yuqoridagi keltirilgan formulalarga doir ba'zi namunaviy misollar ko'ramiz.

1-masala. Ifodani soddalashtiring: $cos^2\alpha - 1 + tg^2\alpha * cos^2\alpha$

Yechim:

$cos^2\alpha - 1 + tg^2\alpha * cos^2\alpha = cos^2\alpha - sin^2\alpha - cos^2\alpha + \frac{sin^2\alpha}{cos^2\alpha} * cos^2\alpha =$

$-sin^2\alpha + sin^2\alpha = 0$

2-masala. Ifodani soddalashtiring: $cosx * tgx + cosx * ctgx$

Yechim:

$cosx * tgx + cosx * ctgx = cosx * \frac{sinx}{cosx} + cosx * \frac{cosx}{sinx} = sinx + \frac{cos^2x}{sinx} = \frac{1}{sinx}$

3-masala. $\frac{cos2\alpha}{cos\alpha - sin\alpha}$ ning qiymatini hisoblang.

Yechim:

$\frac{cos2\alpha}{cos\alpha - sin\alpha} = \frac{cos^2\alpha - sin^2\alpha}{cos\alpha - sin\alpha} = \frac{(cos\alpha - sin\alpha)(cos\alpha + sin\alpha)}{cos\alpha - sin\alpha} = cos\alpha + sin\alpha$

4-masala. $\sin x = \frac{1}{4}$ $berilgan\ bo'lsa$, $\cos 5x * \cos 7x + \sin 5x * \sin 7x$ ni hisoblang.

Yechim:

$\cos 5x * \cos 7x + \sin 5x * \sin 7x = \cos(7x - 5x) = \cos 2x$; $\sin x = \frac{1}{4}$ masala shartidan foydalanamiz.

$\cos 2x = cos^2x - sin^2x = 1 - sin^2x - sin^2x = 1 - 2sin^2x = 1 - 2 * \left(\frac{1}{4}\right)^2 =$

$= 1 - \frac{2}{16} = \frac{7}{8}$

5-masala. Ifodani soddalashtiring: $\cos\left(\frac{\pi}{2} - \alpha\right) + \sin(\pi - \alpha)$

Yechim:

$$\cos\left(\frac{\pi}{2} - \alpha\right) + \sin(\pi - \alpha) = -\sin\alpha + \sin\alpha = 0$$

6-masala. Ifodani soddalashtiring: $\frac{\cos(\alpha+\beta)+2\sin\alpha\cos\beta}{\cos(\alpha-\beta)}$

Yechim:

$$\frac{\cos(\alpha+\beta) + 2\sin\alpha\cos\beta}{\cos(\alpha-\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta + 2\sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} = 1$$

7-masala. Ifodaning eng kichik qiymatini hisoblang: $2\sin\alpha\cos\alpha\,\mathrm{tg}\,\alpha + 1$

Yechim:

$2\sin\alpha\cos\alpha\,\mathrm{tg}\,\alpha + 1 = 2\sin\alpha\cos\alpha\frac{\sin\alpha}{\cos\alpha} + \cos^2\alpha + \sin^2\alpha = \cos^2\alpha + +3\sin^2\alpha =$

$1,\ \alpha = \pi k$

8-masala. Ifodaning eng katta qiymatini hisoblang: $3 - 2\sin\alpha\cos\alpha\,\mathrm{ctg}\,\alpha$

Yechim:

$3 - 2\sin\alpha\cos\alpha\,\mathrm{ctg}\,\alpha = 3\cos^2\alpha + 3\sin^2\alpha\text{-}2\sin\alpha\cos\alpha\frac{\sin\alpha}{\cos\alpha} = \cos^2\alpha +$

$+3\sin^2\alpha = 1 + 2\cos^2\alpha = 3,\quad \alpha = \pi k$

9-masala: $ctg\left(\frac{3\pi}{2} + \alpha\right) + ctgx = 5$ tenglik o'rinli bo'lsa, u holda $tg^2x + ctg^2x$ ning qiymatini toping.

Yechim:

$$(-tgx + ctgx)^2 = 5^2$$
$$tg^2x - 2tgxctgx + ctg^2x = 25$$
$$tg^2x - 2 + ctg^2x = 25$$
$$tg^2x + ctg^2x = 27$$

10-masala. Ifodaning qiymatini hisoblang: $\frac{sin^4\alpha+cos^4\alpha-1}{sin^6\alpha+cos^6\alpha-1}$

Yechim:

$$\frac{sin^4\alpha + cos^4\alpha - 1}{sin^6\alpha + cos^6\alpha - 1} = \frac{sin^4\alpha + cos^4\alpha - sin^2\alpha - cos^2\alpha}{sin^6\alpha + cos^6\alpha - sin^2\alpha - cos^2\alpha} =$$

$$= \frac{sin^2\alpha(-1 + sin^2\alpha) + cos^2\alpha(cos^2\alpha - 1)}{sin^2\alpha(sin^4\alpha - 1) + cos^2\alpha(cos^4\alpha - 1)} =$$

$$= \frac{-sin^2\alpha cos^2\alpha - cos^2\alpha sin^2\alpha}{sin^2\alpha(sin^4\alpha - sin^2\alpha - cos^2\alpha) + cis^2\alpha(cos^4\alpha - cos^2\alpha - sin^2\alpha)} =$$

$$= \frac{-2sin^2\alpha cos^2\alpha}{sin^2\alpha(sin^2\alpha(sin^2\alpha - 1) - cos^2\alpha) + cos^2\alpha(cos^2\alpha(cos^2\alpha - 1) - sin^2\alpha)} =$$

$$= \frac{-2sin^2\alpha cos^2\alpha}{sin^2\alpha(-sin^2\alpha cos^2\alpha - cos^2\alpha) + cos^2\alpha(-sin^2\alpha cos^2\alpha - sin^2\alpha)} =$$

$$= \frac{-2sin^2\alpha cos^2\alpha}{-sin^2\alpha cos^2\alpha(sin^2\alpha + 1) - sin^2\alpha cos^2\alpha(cos^2\alpha + 1)} =$$

$$= \frac{-2sin^2\alpha cos^2\alpha}{-sin^2\alpha cos^2\alpha(sin^2\alpha + cos^2\alpha + 2)} = \frac{2}{3}$$

11-masala. Ifodani soddalashtiring: $sin^3 2\alpha cos6\alpha + cos^6\alpha sin6\alpha$

Yechim:

$$sin^3 2\alpha cos6\alpha + cos^6\alpha sin6\alpha = (sin2\alpha)^3 cos2 \cdot 3\alpha + (cos2\alpha)^3 \alpha sin2 \cdot 3\alpha$$

$$= sin2\alpha cos2\alpha((sin2\alpha)^2 cos2 \cdot 2\alpha + (cos2\alpha)^2 sin2 \cdot 2\alpha) =$$

$$= \frac{1}{2} sin4\alpha\big(sin2\alpha cos2\alpha(sin2\alpha cos2\alpha + sin2\alpha cos2\alpha)\big) =$$

$$= \frac{1}{2} sin4\alpha(cos2\alpha sin4)\alpha = \frac{1}{4} sin4\alpha \cdot sin4\alpha \cdot sin4\alpha = \frac{1}{4} sin^3 4\alpha$$

Yuqorida keltirilib o'tilgan namunaviy masalalar maktab o'quvchilari va oliy ta'lim muassasalari talabalari uchun trigonometrik funksiyalar va ularni ayniy almashtirish borasida elementar tushunchalarni paydo qilishda muhim ahamiyat kasb etadi.

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