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GRONUOLL CHEGARALANISHLI IKKINCHI TARTIBLI BOSHQARUVLAR UCHUN TUTISH MASALASI

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Annotatsiya. *Boshqaruvlar Granoull chegaralanishga ega holda ikkinchi tartibli differensial o‘yinlar uchun tutish masalasi ushbu ma’ruzada o‘rganiladi.*
Kalit so`zlar: *Parallel quvish strategiyasi, quvlovchi, qochuvchi, differensial o’yin, geometrik chegaralanish, tezlanish, Granoull chegaralanishli.*

GRONWALL IS A CATCH ISSUE FOR BOUNDED SECOND ORDER CONTROLS

Annotation. *The issue of Capture for second order differential games with Granoull delimitation of controls is explored in this lecture.*
Keywords: *Parallel chase strategy, chaser, escapee, differential game, geometric delimitation, acceleration, Granoull bounded.*

Introduction

P va **E** obyektlari $\boldsymbol{R^n}$ fazoda berilgan va ularning harakatlari quyidagi differensial tenglamalarga asoslangan.

$$\mathbf{P}:\ \ddot{x} = u\ ,\ \dot{x}(0) - kx(0) = 0, \quad |u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\,ds\,, \qquad (1)$$

$$\mathbf{E}:\ \ddot{y} = v\,,\dot{y}(0) - ky(0) = 0, \quad |v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\,ds\,, \qquad (2)$$

bu yerda x – **P** obyektning $\boldsymbol{R^n}$ fazodagi holati, $x_0 = x(0)$, $x_1 = \dot{x}(0)$–uning mos ravishda $t = 0$ vaqtdagi boshlang‘ich holati va boshlang‘ich tezligi; u – quvlovchining boshqariladigan tezlanishi bo‘lib $u:[0,\infty) \to \boldsymbol{R^n}$ va u vaqt bo‘yicha o‘lchanuvchi funksiya sifatida tanlanadi; barcha $|u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds$ shartni qanoatlantiruvchi bunday $u(\cdot)$ o‘lchanuvchi funksiyalar to’plamini $\boldsymbol{G_p}$ bilan belgilaymiz. y–**E** obyektning $\boldsymbol{R^n}$ fazodagi holati, $y_0 = y(0)$, $y_1 = \dot{y}(0)$ – uning mos ravishda barcha $|v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\, ds$ shartni qanoatlantiruvchi bunday $v(\cdot)$ o‘lchanuvchi funksiyalar to‘plamini $\boldsymbol{G_E}$ bilan belgilaymiz.

Research Methods and the Received Results

Ta’rif 1. Agar $(x_0,\ x_1,\ u(\cdot))$, $u(\cdot) \in \boldsymbol{G_p}$ uchlik berilgan bo’lsa, (1) tenglamaning quyidagi yechimiga quvlovchining harakat trayektoriyasi deyiladi

$$x(t) = x_0 + tx_1 + \int_0^t\int_0^s u(\tau)\, d\tau ds.$$

Ta’rif 2. Agar $(y_0,\ y_1, v(\cdot))$, $v(\cdot) \in \boldsymbol{G_E}$ uchlik berilgan bo‘lsa (2) tenglamaning quyidagi yechimiga qochuvchining harakat trayektoriyasi deyiladi

$$y(t) = y_0 + ty_1 + \int_0^t\int_0^s v(\tau)\, d\tau ds.$$

Ta’rif 3. (1)-(2) masala uchun tutish masalasi ([1]-[2]) yechilgan deyiladi, agar qochuvchining ixtiyoriy $v(\cdot) \in \boldsymbol{G_E}$ boshqaruv funksiyasi uchun quvlovchining shunday $u^*(\cdot) \in \boldsymbol{G_p}$ boshqaruv funksiya mavjud bo‘lsaki, biror chekli t^* vaqtda quyidagi tenglik bajarilsin

$$x(t^*) = y(t^*). \tag{3}$$

Ta’rif 4. Quvlovchining (1) – (2) masala uchun П-strategiyasi ([3]-[4]) deb quyidagi funksiyaga aytamiz,

$$u(v) = v - \lambda(v)\xi_0, \tag{4}$$

bunda $\xi_0 = \dfrac{z_0}{|z_0|}$, $\lambda(v) = (v,\xi_0) + \sqrt{(v,\xi_0)^2 + \delta e^{2lt}}$, $\delta = \rho^2 - \sigma^2 \ge 0$, (v,ξ_0) – v va ξ_0 vektorlarning $\boldsymbol{R^n}$ fazodagi skalyar ko’paytmasi.

Teorema. Agar Granoull chegaralanishli ikkinchi tartibli differensial o‘yin (1)-(2) uchun quyidagi shart $\rho > \sigma$ o‘rinli bo‘lsa, u holda П-strategiya (4) yordamida tutish masalasi (0,t) yechiladi va obyektlar orasidagi yaqinlashish funksiyasi quyidagicha bo‘ladi

$$f(l,t,|z_0|,\rho,\sigma,k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

Conclusion

Isboti. Agar qochuvchi ixtiyoriy $v(\cdot) \in \boldsymbol{G_E}$ bo‘lganda, quvlovchi esa (4) ko‘rinishdagi strategiyani tanlasin, faraz qilamiz, u holda (1) va (2) tenglamalarga asosan quyidagi Karateodori tenglamasini topamiz.

$$\ddot{z} = -\lambda\left(v(t)\right)\xi_0, \qquad \dot{z}(0) - kz(0) = 0,$$

Quyidagi yechim boshlang’ich shartlarni berilishi bo’yicha bundan aniqlanadi

$$z(t) = z_0\left(kt+1\right) - \xi_0 \int_0^t\int_0^s \lambda\left(v(\tau),\xi_0\right)d\tau ds$$

yoki

$$|z(t)| = |z_0|\left(kt+1\right) - \int_0^t\int_0^s ((v,\xi_0) + \sqrt{(v,\xi_0)^2 + \delta e^{2lt}})d\tau ds\,.$$

Lemmaga ko’ra quyidagi tengsizliklarni hosil qilamiz

$$|z(t)| \le |z_0|\left(kt+1\right) - \int_0^t\int_0^s e^{l\tau}(\rho-\sigma)d\tau ds \Rightarrow$$

$$|z(t)| \le |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

Agar $f(l,t,|z_0|,\rho,\sigma,k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$ desak

bu funksiyani nolga aylantiruvchi musbat t^* vaqtni topamiz.

$$\frac{\rho-\sigma}{l^2}e^{lt} = |z_0|(kt+1) + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t\,,$$

oxirgi tenglikni soddalashtirish orqali quyidagi tenglikni hosil qilamiz,

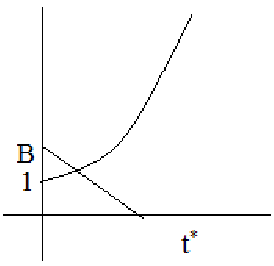
$$e^{lt}=t\left(\frac{|z_0|kl^2}{\rho-\sigma}+l\right)+\frac{|z_0|l^2}{\rho-\sigma}+1$$

bunda $A=\frac{|z_0|kl^2}{\rho-\sigma}+l$, $B=\frac{|z_0|l^2}{\rho-\sigma}+1$ bo‘lib, bu yerda $\rho>\sigma$, $B>1$. Natijada quyidagi tenglikka ega bo‘lamiz

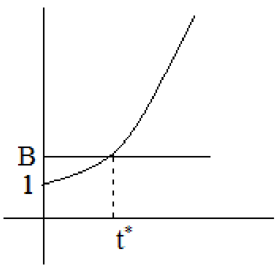
$$e^{lt}=At+B \qquad (5)$$

Tutish vaqtini aniqlash uchun (5) tenglamani quyidagi hollarini ko‘rib chiqamiz.

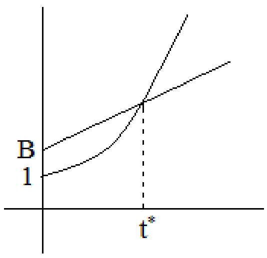
1. $A<0 \;\Rightarrow\; k<\frac{\sigma-\rho}{|z_0|l}$ bo‘lsin. U holda (5) tenglama yagona $t^*>0$ musbat yechim mavjud va bu yechim tutish vaqti bo‘ladi. (1-chizma)
2. $A=0 \;\Rightarrow\; k=\frac{\sigma-\rho}{|z_0|l}$ bo‘lsin. U holda (5) tenglama yechimi $t^*=\frac{\ln\left(\frac{|z_0|l^2}{\rho-\sigma}+1\right)}{l}$ bo‘lib, tutish vaqtini beradi.
3. $A>0 \Rightarrow k>\frac{\sigma-\rho}{|z_0|l}$ bo‘lsin. U holda (5) tenglama $t^*>0$ musbat yechimi mavjud va bu yechim tutish vaqti bo‘ladi.



(1-chizma)



(2-chizma)



(3-chizma)

SUMMARY

Differensial o'yinlar nazariyasida quvish va qochish masalalari aloxida alohida o'rin tutadi. Ulardan biri turli usullarni amalga oshirishning kengligi, shuningdek

olingan natijalarning o'ziga xosligi. Bu xususiyat, ayniqsa, model savollarida yaqqol ko'rinadi. Lemmada berilgan shartga muvofiq teorema shartlangan va isbot beradi. Differentsial o'yinlar nazariyasida boshqaruv elementlariga geometrik, integral va ularning birgalikdagi cheklovlari qo'yilgan savollar etarlicha o'rganilgan. Ushbu maqola gromvel lemmasidan foydalangan holda delimitatsiya deb nomlangan boshqaruv funktsiyasidagi yangi nazorat sinflarini o'z ichiga oladi. Ikkinchi darajali differentsial o'yinda quvish-qochish muammosi o'rganildi va ta'qibchi va qochuvchi uchun yangi etarlilik shartlari taklif qilindi.

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