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MATEMATIKA-FIZIKA FANIDAN ISSIQLIK TARQALISH TENGLAMASI UCHUN ARALASH MASALALARNI FUR’E USULI BILAN YECHISH

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Annotatsiya. *Ushbu maqolada talabalarning kasbiy madaniyatini rivojlantirish muammosining dolzarbligi va nazariy asoslari, maktab matematika o‘qituvchilarining kasbiy madaniyatini rivojlantirish jarayoni, matematik-fizika fanidan masalalar yechish, reflektiv va ijodiy fikrlash, masala tuzilmalar, tarkibiy qismlarning mazmuni ta’kidlanadi.*

Аннотация. *В данной статье освещены актуальность и теоретические основы проблемы развития профессиональной культуры учащихся, процесс развития профессиональной культуры школьных учителей математики, решения задач по математической физике, рефлексивного и творческого мышления, структура проблем, содержание компоненты.*

Annotation*. This article highlights the relevance and theoretical foundations of the problem of developing the professional culture of students, the process of developing the professional culture of school mathematics teachers, solving problems in mathematical physics, reflective and creative thinking, the structure of problems, the content of components.*

Renessans Ta’lim Universitetida fanlarni o‘qitishda zamonaviy pedagogik texnologiya, fanlararo aloqadorlikdan foydalanish va amaliy mashg‘ulotlarni o‘tkazish davomida talabalarning faoliyatida o‘qitilayotgan matematik-fizika tenglamalari fani mavzulariga doir nazariy va amaliy masalalar yechimlarini o‘rganishni tashkil etadi.

Issiqlik tarqalish tenglamasi uchun aralash masalalarni Fur'e usuli bilan yechish namunalari:

1-masala. $\Omega=\{(x,t): 0 < x<l, 0<t<+\infty\}$ sohada $u_t = a^2 u_{xx}$ tenglamaning

$$u(x,0)=\begin{cases} x, & 0<x\le l/2; \\ l-x, & \ell/2\le x<l \end{cases}$$

boshlang'ich va *u(0,t)=u(l,t)=0* chegaraviy shartlarni qanoatlantiruvchi yechimi topilsin.

Yechilishi: Masala yechimini (8) qator ko'rinishda izlaymiz. Bu qatorning koeffitsientini (9) formula yordamida topamiz:

$$a_n=\frac{2}{l}\int_0^l \varphi(x)\sin\frac{\pi n}{l}xdx=\frac{2}{l}\left\{\int_0^{l/2} x\sin\frac{\pi n}{l}xdx+\int_{l/2}^{l}(l-x)\sin\frac{\pi n}{l}xdx\right\}$$

Ikkinchi integralda *l-x=y* almashtirish bajarib, ba'zi hisob-kitoblardan keyin, *y* ni yana *x* bilan almashtirib, ushbu

$$a_n=\frac{2}{l}\left[1-(-1)^n\right]\int_0^{l/2} x\sin\frac{\pi n}{l}xdx$$

tenglikka ega bo'lamiz. Bo'laklab itegrallash natijasida ushbuni topamiz:

$$a_n=2\left[1-(-1)^n\right]\frac{l}{\pi n}\left\{-\frac{1}{2}\cos\frac{\pi n}{2}+\frac{1}{\pi n}\sin\frac{\pi n}{2}\right\}.$$

Topilgan a_n koeffitsientning qiymatini (8) qatorga qo'yib, masala yechimini hosil qilamiz:

$$u(x,t)=\frac{2l}{\pi}\sum_{n=1}^{\infty}[1-(-1)^n]\left(-\frac{1}{2n}\cos\frac{\pi n}{2}+\frac{1}{\pi n^2}\sin\frac{\pi n}{2}\right)e^{-\left(\frac{\pi n}{l}a\right)^2 t}\sin\frac{\pi n}{l}x.$$

Agar $n=2k$ bo‘lsa, $1-(-1)^n=0$, agar $n=2k+1$ bo‘lsa, $1-(-1)^n=2$ va $\cos\frac{\pi n}{2}=\cos(\pi k+\frac{\pi}{2})=0$, $\sin\frac{\pi n}{2}=\sin(\pi k+\frac{\pi}{2})=(-1)^k$ bo‘lganligi uchun yechimni quyidagi ko‘rinishda yozish mumkin:

$$u(x,t)=\frac{4l}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}e^{-\left(\frac{\pi(2k+1)}{l}a\right)^2 t}\sin\frac{\pi(2k+1)}{l}x.$$

2-masala. $\Omega=\{(x,t): 0<x<1, 0<t<+\infty\}$ sohada

$$u_t-u_{xx}=t(x+1) \qquad (23)$$

tenglamaning

$$u(x,0)=0,\ \ 0\le x\le 1 \qquad (24)$$

boshlang‘ich va

$$u_x(0,t)=t^2,\ \ u(1,t)=t^2 \qquad (25)$$

chegaraviy shartlarni qanoatlantiruvchi yechimi topilsin.

Yechilishi: Berilgan masalada $a=1$, $l=1$, $\varphi(x)=0$ bo‘lib, (25) chegaraviy shartlarda noma’lum $u(x,t)$ funksiyaning hosilasi qatnashganligi hamda bu shartning bir jinsli bo‘lmaganligi sababli bu masalani quyidagicha yechamiz.

Masala echimini $u(x,t)=\omega(x,t)+z(x,t)$ ko‘rinishda izlaymiz, bunda $\omega(x,t)$ funksiyani ushbu $\omega(x,t)=A(t)x+B(t)$ ko‘rinishda izlab, (25) chegaraviy shartlardan $A(t)=t^2$, $B(t)=0$ bo‘lishini, hamda $\omega(x,t)=xt^2$ ekanligini topamiz.

U holda

$$z(x,t)=u(x,t)-xt^2 \qquad (26)$$

funksiya

$$z_t-z_{xx}=(1-x)t \qquad (27)$$

tenglamani va

$$z(x,0)=0,\quad z_x(0,t)=z(1,t)=0 \qquad (28)$$

shartlarni qanoatlantiruvchi aralash masalaning yechimi bo‘ladi.

(1)-(3) aralash masalani yechishdagi kabi z_t-z_{xx}=0 bir jinsli tenglamaning (28) dagi chegaraviy shartlarni qanoatlantiruvchi yechimini *z(x,t)=X(x)T(t)* ko‘rinishda izlab,

$$X''(x)+\lambda^2 X(x)=0,$$
$$X'(0)=X(1)=0$$

Shturm-Liuvill masalasiga kelamiz. Bu masalaning xos sonlari $\lambda_n=\dfrac{\pi}{2}+\pi n,\quad n=0,1,2,3,\dots$ va bularga mos trivial bo‘lmagan xos funksiyalari $X_n(x)=c'sλ_n x$ ko‘rinishda ekanligini topamiz.

U holda (27), (28) masalaning yechimini

$$z(x,t)=\sum_{n=0}^{\infty}P_n(t)X_n(x)=\sum_{n=0}^{\infty}P_n(t)\cos\lambda_n x \tag{29}$$

ko‘rinishda izlaymiz va uni (27) tenglamaga qo‘yib,

$$\sum_{n=0}^{\infty}\left[T_n'(t)+\lambda_n^2T_n(t)\right]\cos\lambda_n x=(1-x)t \tag{30}$$

tenglikni hosil qilamiz. *1-x* funksiyani $X_n(x)=\cos\lambda_n x$ xos funksiyalar sistemasi bo‘yicha (0;1) intervalda Fure qatoriga yoyamiz.

$$1-x=\sum_{n=0}^{\infty}a_n\cos\lambda_n x$$
$$a_n=2\int_0^1(1-x)\cos\lambda_n x dx=\frac{2}{\lambda_n^2} \tag{31}$$

U holda (30) va (31) ni taqqoslab, noma’lum $T_n(t)$ funksiyalarga nisbatan

$$T_n'(t)+\lambda_n^2T_n(t)=\frac{2t}{\lambda_n^2},\quad n=0,1,2,3,\dots \tag{32}$$

differensial tenglamalarni hosil qilamiz.

(32) tenglamaning $T_n(0)=0$ boshlang‘ich shartni qanoatlantiruvchi yechimi

$$T_n(t)=2\lambda_n^{-6}\left(e^{-\lambda_n^2 t}+\lambda_n^2 t-1\right) \tag{33}$$

ko‘rinishda bo‘ladi.

Shunday qilib, (26), (29) va (33) ga asosan (23)-(25) aralash masalaning yechimi

$$u(x,t) = xt^2 + 2\sum_{n=0}^{\infty} \lambda_n^{-6}\left(e^{-\lambda_n^2 t} + \lambda_n^2 t - 1\right)\cos\lambda_n x$$

ko'rinishda ekanligini topamiz, bu yerda $\lambda_n = \frac{\pi}{2} + \pi n$.

3-masala. $\Omega=\{(x,t): 0< x < 1, 0 < t < +\infty\}$ cohada $u_t=u_{xx}-2u_x+x+2t$, $u(x,0)=e^x \sin \pi x$, $u(0,t)=0$, $u(1,t)=t$, aralash masalaning yechimi topilsin.

***Yechilishi*:** Berilgan masalada

$a=1$, $l=1$, $b=-2$, $c=0$, $F(x,t)= x+2t$, $\varphi(x)=e^x \sin\pi x$, $\mu_1(t)=0$, $\mu_2(t)=t$ chegaraviy shart bir jinsli bo'lmaganligi sababli masala yechimini

$$u(x,t) = z(x,t) + \omega(x,t) \tag{34}$$

ko'rinishda izlaymiz. Bu yerda $\omega(x,t)$ yordamchi funksiya bo'lib, uni faqat, chegaraviy shartlarni qanoatlantiradigan qilib tanlaymiz.

$\omega(x,t) = \mu_1(t) + \frac{x}{2}[\mu_2(t) - \mu_2(t)]$ ga asosan $\omega(x,t)=xt$ bo'ladi.

U holda $z(x,t) = u(x,t)-xt$ funksiya uchun quyidagi $z_t=z_{xx}-2z_x$,

$$z(x,0)=e^x \sin\pi x,\quad z(0,t) = z(1,t)=0$$

aralash masalaga kelamiz. Bu masalada

$$z(x,t) = e^{x-t} v(x,t) \tag{35}$$

almashtirish bajarsak $v(x,t)$ noma'lum funksiyaga nisbatan ushbu

$$v_t = v_{xx}, \tag{36}$$

$$v(x,0)= e^{-x} z(x,0) = \sin \pi x,\quad v(0,t)= v(1,t)=0 \tag{37}$$

aralash masala hosil bo'ladi. Bu masala yechimini (8) qator ko'rinishida izlaymiz va uning koeffitsientini (9) formula yordamida topamiz:

$$a_n = 2\int_0^1 \sin\pi x \sin\pi n x dx = \begin{cases} 0, & агар \quad n \neq 1, \\ 1, & агар \quad n = 1. \end{cases}$$

Demak, $a_n=0$, $n\neq 1$ bo'lsa va $a_1=1$.

U holda (36)-(37) masalaning yechimi

$$v(x,t) = e^{-\pi^2 t}\sin\pi x \tag{38}$$

ko‘rinishda bo‘ladi.

Shunday qilib (34), (35) va (38) ga asosan berilgan masalani yechimi

$$u(x,t) = xt + \sin \pi x \cdot e^{x-t-\pi^2 t}$$

ko‘rinishda ekanligini topamiz.

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