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# DIFFUZIYA TENGLAMASINING FUNDAMENTAL YECHIMI VA UNING XOSSALARI

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***ANNOTATSIYA***

*Mazkur maqolada parabolik tipdagi asosiy tenglamalardan biri bo‘lgan diffuziya tenglamasi uchun fundamental yechimi va uning xossalari bayon qilinadi. Fundamental yechimining beshta xossasi keltirilib va har bir xossaning isboti ko‘rsatiladi.*

***Kalit so‘zlar:*** *Diffuziya tenglamasi, qo‘shma tenglama, fundamental yechim, Lopital teoremasi.*

## KIRISH

Fundamental yechim parabolik tenglamaning muhim yechimlaridan biridir. U odatda boshlang‘ich shartlar yoki chegaraviy shartlariga bog‘liq bo‘lmagan, delta funktsiyasi sifatida berilgan boshlang‘ich shartlar bilan tenglama yechimini topishda ishlatiladi.

Fundamental yechim yordamida diffuziya tenglamasining har qanday boshlang‘ich shartlari uchun umumiy yechimlarni qurish mumkin. Masalan, agar boshlang‘ich shartlar $u(x,0)$ biror funksiyaga teng bo‘lsa, u holda bu funksiyani fundamental yechim bilan konvolyutsiya qilish orqali umumiy yechimni olish mumkin. Fundamental yechimlar matematik nazariyalar va fizika masalalarida keng qo‘llaniladi. Ular yordamida haqiqiy fizik muammolarni soddalashtirish va yechimlarni topish mumkin.

Fundamental yechimlar muayyan muammolarni chuqurroq tahlil qilish uchun zarurdir. Ular yordamida tenglamaning sifatli va miqdoriy xususiyatlarini aniqlash mumkin [1,2,6].

Umuman olganda, fundamental yechimlar parabolik tenglamalarning yechimlari haqida chuqurroq tushuncha beradi va bu yechimlarning xususiyatlarini o'rganishda muhim rol o'ynaydi.

## ASOSIY QISM

Odatda ikki juft argumentga bog'liq bo'lgan ushbu

$$E(x,t;\xi,\eta)=\frac{1}{2\sqrt{\alpha(t-\eta)}}exr[-\frac{(x-\xi)^2}{4(t-\eta)}],\quad t>\eta \qquad (1)$$

funksiya

$$u_t(x,t)=\lambda u_{xx}(x,t),\quad (x,t)\in D \qquad (2)$$

tenglamaning fundamental yechimi deyiladi. Bu yerda $\lambda$-diffuziya koeffitsiyenti, $D$ esa $\{(x,t):\ 0<x<L,\ 0<t\le T\}$ sohani ifodalaydi. [2,3,4]

$E(x,t;\xi,\eta)$ funksiya quydagi xossalarga ega:

1°. $x$ va $t$ o'zgaruvchilar bo'yicha (2) tenglamani qanoatlantiradi.

**Isbot.** Bevosita hisoblashlar ko'rsatadiki,

$$\frac{\partial E}{\partial t}=\left[\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{\frac{5}{2}}}-\frac{1}{4\sqrt{\pi}(t-\eta)^{\frac{3}{2}}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right],$$

$$\frac{\partial E}{\partial x}=\frac{x-\xi}{4\sqrt{\Pi}(t-\eta)^{\frac{3}{2}}}exr\left[-\frac{(x-\xi)^2}{4(y-\eta)}\right],$$

$$\frac{\partial^2 E}{\partial x^2}=\left[\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{\frac{5}{2}}}-\frac{1}{4\sqrt{\pi}(t-\eta)^{\frac{3}{2}}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right].$$

Bulardan darxol $E_{xx}-E_t=0$ tenglik kelib chiqadi.

2°. $\xi,\ \eta$ o'zgaruvchilar bo'yicha (2) ga qo'shma bo'lgan $\upsilon_{\xi\xi}+\upsilon_\eta=0$ tenglamani qanoatlantiradi.

**Isbot.** Xaqiqatan ham

$$\frac{\partial E}{\partial \eta}=\left[\frac{1}{4\sqrt{\pi}(t-\eta)^{\frac{3}{2}}}-\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{\frac{5}{2}}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right],$$

$$\frac{\partial E}{\partial \xi}=\frac{x-\xi}{4\sqrt{\Pi}(t-\eta)^{\frac{3}{2}}}exr\left[-\frac{(x-\xi)^2}{4(y-\eta)}\right],$$

$$\frac{\partial^2 E}{\partial \xi^2}=\left[\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{\frac{5}{2}}}-\frac{1}{4\sqrt{\pi}(t-\eta)^{\frac{3}{2}}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right].$$

bo‘lib, bulardan darxol $E_{\xi\xi}+E_{\eta}=0$ tenglik kelib chiqadi.

3°. $x\neq\xi$ bo‘lganda $\lim\limits_{\eta\to t}E(x,t;\xi,\eta)=0$ tenglik o‘rinli.

**Isbot.** Bu xossani isbotlash maqsadida $E(x,t;\xi,\eta)$ funksiyani

$$E(x,t;\xi,\eta)=\left\{\frac{1}{2\sqrt{\pi(t-\eta)}}/\,exr[\frac{(x-\xi)^2}{4(t-\eta)}]\right\}$$

ko‘rinishda yozsak $\eta\to t$ da $\frac{\infty}{\infty}$ aniqmaslikka ega bo‘lamiz. Shuning uchun Lopital teoremasiga ko‘ra

$$\lim_{\eta\to t}E(x,t;\xi,\eta)=$$

$$\lim_{\eta\to t}\left\{\frac{\dfrac{1}{4\sqrt{\pi}(t-\eta)^{\frac{3}{2}}}}{\dfrac{(x-\xi)^2}{4(t-\eta)^2}\exp\left[\dfrac{(x-\xi)^2}{4(t-\eta)}\right]}\right\}=\frac{1}{\sqrt{\pi}}\lim_{\eta\to t}\left\{\frac{\sqrt{t-\eta}}{(x-\xi)^2}\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]\right\}=0$$

4°. $\forall\varphi(x)\in C[0,l]$ funksiya uchun quyidagi tenglik o‘rinli:

$$\lim_{\eta\to t}\int_0^l E(x,t;\xi,\eta)\varphi(\xi)d\xi=\begin{cases}\frac{1}{2}\varphi(0), agar x=0 bo'lsa\\ \varphi(x), agar x\in(0,l) bo'lsa\\ \frac{1}{2}\varphi(l), agar x=l bo'lsa\end{cases}\qquad(3)$$

**Isbot.** (1.13) tengsizlikning chap tomonidagi integralda.

$$\frac{x-\xi}{2\sqrt{t-\eta}}=\xi \qquad \text{yani} \qquad \xi=x-2\xi\sqrt{t-\eta}$$

almashtirish bajaramiz. U holda $d\xi=-2\sqrt{t-\eta}\,d\xi$ va quyidagi

$$\xi=0\Rightarrow\xi=\frac{x}{2\sqrt{t-\eta}},\qquad \xi=l\Rightarrow\zeta=\frac{x-l}{2\sqrt{t-\eta}}.$$

tengsizliklar o‘rinli bo‘lgan. Bularni inobatga olsak,

$$\int_0^l E(x,t;\xi,\eta)\phi(\xi)d\xi=\frac{1}{\sqrt{\pi}}\int_{(x-l)/2\sqrt{t-\eta}}^{x/2\sqrt{t-\eta}}e\phi(x-2\varsigma\sqrt{t-\eta})d\varsigma \qquad (4)$$

tenglikka ega bo‘lamiz.

Matematik anializdan malum bo‘ladi

$$\int_{-\infty}^{+\infty} e^{-\varsigma^2} d\varsigma = 2\int_{0}^{+\infty} e^{-\varsigma^2} d\varsigma = 2\int_{-\infty}^{0} e^{-\varsigma^2} d\varsigma = \sqrt{\pi}$$

tengliklarni etiborga olib, (4) tenglikda $\eta$ ni $t$ ga intiltirib, limitga o‘tamiz:

$$\lim_{\eta\to t}\int_0^l E(x,t;\xi,\eta,)\varphi(\xi)d\xi = \frac{1}{\sqrt{\pi}}\lim_{\eta\to t}\int_{(x-l)/2\sqrt{t-\eta}}^{x/2\sqrt{t-\eta}} e^{-\zeta^2}\varphi(x-2\zeta\sqrt{t-\eta})d\zeta =$$

$$=\begin{cases} \dfrac{1}{\sqrt{\pi}}\varphi(0)\displaystyle\int_{-\infty}^{0} e^{-\zeta^2}d\zeta = \dfrac{1}{2}\varphi(0),\ agar\ x=0\ bo'lsa \\ \dfrac{1}{\sqrt{\pi}}\varphi(x)\displaystyle\int_{-\infty}^{+\infty} e^{-\zeta^2}d\zeta = \varphi(x)\ agar\ x\in(0,l)\ bo'lsa \\ \dfrac{1}{\sqrt{\pi}}\varphi(l)\displaystyle\int_{0}^{+\infty} e^{-\zeta^2}d\zeta = \dfrac{1}{2}\varphi(l),\ agar\ x=l\ bo'lsa. \end{cases}$$

$$5^\circ.\ \lim_{\eta\to t}\int_0^l E(x,t;\xi,\eta)\varphi(\xi)d\xi = \begin{cases} \dfrac{1}{2},\ agar\quad x=0\quad bo'lsa; \\ 1,\ agar\ x\in(0,l)\ bo'lsa; \\ \dfrac{1}{2},\ agar\quad x=l\quad bo'lsa. \end{cases} \qquad (5)$$

(5) tenglikning to‘g‘riligi (3) tenglikdan $\varphi(t)\equiv\ t\in[0,l]$ bo‘lganda darhol kelib chiqadi.

**Foydalanilgan adabiyotlar.**

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